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Intellectual
Mathematics

Teacher's Manual

Class VIII

Written by :
Author's Team
(Vidyalaya Prakashan)

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Content

Vrindavan Books International

New Delhi

Sales Office :

C-24, Jwala Nagar, Transport Nagar, Meerut - 250002

Ph. : 0121 - 2400630, 8899271392

Head Office :

A-102, Chandar Vihar, Delhi - 110092

e-mail : vidyalayaprakashan@yahoo.in

1 Number System

Rational Numbers

Exercise 1A

1. (a) $\frac{3}{4}=\frac{3\times2}{4\times2}=\frac{6}{8};\frac{3\times3}{4\times3}=\frac{9}{12};\frac{3\times4}{4\times4}=\frac{12}{16}$

   (b) $\frac{-5}{7}=\frac{-5\times2}{7\times2}=\frac{-10}{14};\frac{-5\times3}{7\times3}=\frac{-15}{21};\frac{-5\times4}{7\times4}=\frac{-20}{28}$

   (c) $\frac{8}{-9}=\frac{8\times2}{-9\times2}=\frac{16}{-18}$ or $\frac{-16}{18}$ (standard form)

   $\frac{-8\times3}{9\times3}=\frac{-24}{27};\frac{-8\times4}{9\times4}=\frac{-32}{36}$

   (d) $\frac{-11}{12}=\frac{-11\times2}{12\times2}=\frac{-22}{24};\frac{-11\times3}{12\times3}=\frac{-33}{36};\frac{-11\times4}{12\times4}=\frac{-44}{48}$

2. $\frac{21}{-8}$ as a rational number with denominator 24.

   To make the denominator $24=-8\times-3=24$

   So, $\frac{21}{-8}\times\frac{-3}{-3}=\frac{-63}{24}$

3. $\frac{-28}{84}$ as a rational number with numerator 4

   To make the numerator $4=\ -28=\frac{-28}{-7}=4$

   So, $\frac{-28\div(-7)}{84\div-7}=\frac{4}{-12}$

4. Add

   (a) $\frac{3}{5}+\frac{1}{5}=\frac{3+1}{5}=\frac{4}{5}$

   (b) $\frac{-8}{11}+\frac{(-4)}{11}=\frac{-8-4}{11}=\frac{-12}{11}$

(c) $\frac{6}{13}+\frac{(-5)}{13}=\frac{6-5}{13}=\frac{1}{13}$

5. Add

(a) $-\frac{1}{6}+\frac{3}{10}$ LCM of 6 and 10 = 30

So, $\frac{-1\times5}{6\times5}+\frac{3\times3}{10\times3}=\frac{-5}{30}+\frac{9}{30}=\frac{-5+9}{30}=\frac{4}{30}$

(b) $\frac{-5}{16}+\frac{-7}{24}$ LCM of 16 and 24 = 48

$\frac{-5\times3}{16\times3}+\frac{-7\times2}{24\times2}=\frac{-15}{48}-\frac{14}{48}=\frac{-15-14}{48}=\frac{-29}{48}$

(c) $3\frac{3}{4}+4\frac{1}{3}=\frac{15}{4}+\frac{13}{3}$ LCM of 4 and 3 = 12

$\frac{15\times3}{4\times3}+\frac{13\times4}{3\times4}=\frac{45}{12}+\frac{52}{12}=\frac{45+52}{12}=\frac{97}{12}=8\frac{1}{12}$

6. (a) $\frac{13}{14}+\frac{9}{-7}$ LCM of 14 and 7 is = 14

$\frac{13}{14}-\frac{9}{7}=\frac{13\times1}{14\times1}-\frac{9\times2}{7\times2}=\frac{13}{14}-\frac{18}{14}=\frac{13-18}{14}=\frac{-5}{14}$

(b) $\frac{6}{15}+\frac{-7}{24}=\frac{6}{15}-\frac{7}{24}$ LCM of 15 and 24 is 120

$\frac{6\times8}{15\times8}-\frac{7\times5}{24\times5}=\frac{48}{120}-\frac{35}{120}=\frac{48-35}{120}=\frac{13}{120}$

(c) $\frac{-16}{9}+\frac{-5}{12}=\frac{-16}{9}-\frac{5}{12}$ LCM of 9 and 12 is 36

$\frac{-16\times4}{9\times4}-\frac{5\times3}{12\times3}=\frac{-64}{36}-\frac{15}{36}=\frac{-64-15}{36}=\frac{-79}{36}$

(d) $\frac{7}{9}+\frac{3}{-4}=\frac{7}{9}-\frac{3}{4}$ LCM of 9 and 4 is 36

$\frac{7\times4}{9\times4}-\frac{3\times9}{4\times9}=\frac{28}{36}-\frac{27}{36}=\frac{28-27}{36}=\frac{1}{36}$

(e) $0+\frac{-3}{5}=\frac{0-3}{5}=\frac{-3}{5}$

7. Additive inverse of

(a) $\frac{3}{5}=\frac{-3}{5}$

(b) $\frac{17}{9}=\frac{-17}{9}$

(c) $\frac{3}{-11}=\frac{3}{11}$

(d) $\frac{8}{-29}=\frac{8}{29}$

(e) $0=0$

(f) $\frac{-11}{-21}=\frac{-11}{21}$

8. Find the sum

(a) $\frac{2}{3}+\frac{-3}{5}+\frac{1}{3}+\frac{-2}{5}$

$$\left(\frac{2}{3}+\frac{1}{3}\right)+\left(\frac{-3}{5}-\frac{2}{5}\right)=\frac{3}{3}-\frac{5}{5}=1-1=0$$

(b) $\frac{3}{5}+\frac{5}{3}+\frac{-11}{5}+\frac{-2}{3}$

$$\left(\frac{3}{5}-\frac{11}{5}\right)+\left(\frac{5}{3}-\frac{2}{3}\right)=\left(\frac{3-11}{5}\right)+\left(\frac{5-2}{3}\right)$$

$$=\frac{-8}{5}+\frac{3}{3}=\frac{-8}{5}+1=\frac{-8+5}{5}=\frac{-3}{5}$$

(c) $\frac{4}{3}+\frac{3}{5}+\frac{-2}{3}+\frac{-11}{5}$

$$\left(\frac{4}{3}-\frac{2}{3}\right)+\left(\frac{3}{5}-\frac{11}{5}\right)=\left(\frac{4-2}{3}\right)+\left(\frac{3-11}{5}\right)$$

$$=\frac{2}{3}-\frac{8}{5}=\frac{10-24}{15}=-\frac{14}{15}$$

(d) $\frac{3}{5}+\frac{7}{3}+\frac{9}{5}+\frac{-13}{15}+\frac{-7}{3}$

$$\left(\frac{3}{5}+\frac{9}{5}\right)+\left(\frac{7}{3}-\frac{7}{3}\right)-\frac{13}{15}=\left(\frac{3+9}{5}\right)+0-\frac{13}{15}$$

$$= \frac{12}{5} - \frac{13}{15} = \frac{36-13}{15} = \frac{23}{15} \text{ or } 1\frac{8}{15}$$

(e) $\frac{5}{3} + \frac{11}{2} + \frac{-9}{4} + \frac{-8}{3} + \frac{-7}{2}$

$$\left(\frac{5}{3} - \frac{8}{3}\right) + \left(\frac{11}{2} - \frac{7}{2}\right) - \frac{9}{4} = \left(\frac{5-8}{3}\right) + \left(\frac{11-7}{2}\right) - \frac{9}{4}$$

$$= \left(\frac{-3}{3}\right) + \left(\frac{4}{2}\right) - \frac{9}{4} = \frac{-1}{1} + \frac{2}{1} - \frac{9}{4}$$

$$= \frac{-4+8-9}{4} = -\frac{-5}{4} \text{ or } -1\frac{1}{4}$$

(f) $\frac{-3}{10} + \frac{7}{15} + \frac{3}{-20} + \frac{-9}{10} + \frac{13}{15} + \frac{-13}{20}$

$$\left(\frac{-3}{10} - \frac{9}{10}\right) + \left(\frac{7}{15} + \frac{13}{15}\right) + \left(\frac{-3}{20} - \frac{13}{20}\right)$$

$$\left(\frac{-3-9}{10}\right) + \left(\frac{7+13}{15}\right) + \left(\frac{-3-13}{20}\right)$$

$$= \frac{-12}{10} + \frac{20}{15} - \frac{16}{20} = \frac{-72+80-48}{60} = \frac{-40}{60} = \frac{-2}{3}$$

Exercise 1B

1. Subtract

(a) $\frac{7}{8} - \frac{2}{3} = \frac{7\times3-2\times8}{24} = \frac{21-16}{24} = \frac{5}{24}$

(b) $\frac{-3}{7} - \left(\frac{-5}{9}\right) = \frac{-3}{7} + \frac{5}{9} = \frac{(-3\times9)+(7\times5)}{63} = \frac{-27+35}{63} = \frac{8}{63}$

(c) $\frac{5}{6} - \frac{3}{4} = \frac{5\times2-3\times3}{12} = \frac{10-9}{12} = \frac{1}{12}$

(d) $\frac{-5}{7} - \left(\frac{-3}{8}\right) = \frac{-5}{7} + \frac{3}{8} = \frac{(-5\times8)+(3\times7)}{56}$

$$= \frac{-40+21}{56} = \frac{-19}{56}$$

2. Evalute

(a) $\frac{3}{5}-\frac{1}{5}=\frac{3-1}{5}=\frac{2}{5}$

(b) $\frac{-1}{6}-\frac{4}{9}=\frac{(-1\times 3)-(4\times 2)}{18}=\frac{-3-8}{18}=\frac{-11}{18}$

(c) $\frac{3}{10}-\left(\frac{-4}{15}\right)=\frac{3}{10}+\frac{4}{15}=\frac{3\times 3+4\times 2}{30}=\frac{9+8}{30}=\frac{17}{30}$

(d) $\frac{4}{5}-\left(\frac{3}{-4}\right)=\frac{4}{5}+\frac{3}{4}=\frac{(4\times 4)+(3\times 5)}{20}=\frac{16+15}{20}=\frac{31}{20}$

3. The sum of rational numbers = –3

One number $=\frac{-11}{5}$

Second number $=\frac{-3}{1}-\left(\frac{-11}{5}\right)=\frac{-3}{1}+\frac{11}{5}$

$=\frac{-3\times 5+11\times 1}{5}=\frac{-15+11}{5}=\frac{-4}{5}$

4. The sum of numbers $=\frac{-3}{5}$

One number $=\frac{-9}{20}$

Other number $=\frac{-3}{5}-\left(\frac{-9}{20}\right)=\frac{-3}{5}+\frac{9}{20}$

$=\frac{(-3\times 4)+(9\times 1)}{20}=\frac{-12+9}{20}=\frac{-3}{20}$

5. Number added should be

$\frac{5}{9}-\left(\frac{-5}{8}\right)=\frac{5}{9}+\frac{5}{8}=\frac{(8\times 5)+(5\times 9)}{72}=\frac{40+45}{72}=\frac{85}{72}$

6. Number added should be

$\frac{5}{9}-\left(\frac{-4}{7}\right)=\frac{5}{9}+\frac{4}{7}=\frac{(5\times 7)+(4\times 9)}{63}=\frac{35+36}{63}=\frac{71}{63}$

7. The number subtracted should be

$$\frac{-7}{11}-\left(\frac{-2}{1}\right)=\frac{-7}{11}+\frac{2}{1}=\frac{-7+22}{11}=\frac{15}{11}$$

8. The number subtracted should be

$$\frac{-3}{4}-\frac{5}{6}=\frac{(-3\times 6)-(5\times 4)}{24}=\frac{-18-20}{24}=\frac{-38}{24}=\frac{-19}{12}$$

9. Simplify :

(a) $\frac{3}{7}+\frac{-6}{11}+\frac{8}{21}+\frac{-5}{22}$

$$\left(\frac{3}{7}+\frac{8}{21}\right)-\left(\frac{6}{11}+\frac{5}{22}\right)=\left(\frac{3\times 3+8\times 1}{21}\right)-\left(\frac{6\times 2+5\times 1}{22}\right)$$

$$=\left(\frac{9+8}{21}\right)-\left(\frac{12+5}{22}\right)=\frac{17}{21}-\frac{17}{22}$$

$$=\frac{17\times 22-17\times 21}{462}=-\frac{374-357}{462}=\frac{17}{462}$$

(b) $\frac{2}{5}+\frac{8}{3}-\frac{12}{15}+\frac{4}{5}-\frac{2}{3}$

$$=\left(\frac{2}{5}+\frac{4}{5}\right)+\left(\frac{8}{3}-\frac{2}{3}\right)-\frac{12}{15}=\frac{6}{5}+\frac{6}{3}-\frac{12}{15}$$

$$=\frac{(6\times 3)+(6\times 5)-(12\times 1)}{15}=\frac{18+30-12}{15}=\frac{48-12}{15}=\frac{36}{15}$$

(c) $\frac{-7}{4}+\frac{5}{3}+\frac{-5}{6}+\frac{1}{3}+\frac{-1}{2}$

$$=\frac{-7}{4}+\left(\frac{5}{3}+\frac{1}{3}\right)-\frac{5}{6}-\frac{1}{2}=\frac{-7}{4}+\frac{6}{3}-\frac{5}{6}-\frac{1}{2}$$

$$=\frac{(-7\times 3)+(6\times 4)-(5\times 2)-(1\times 6)}{12}$$

$$=\frac{-21+24-10-6}{12}=\frac{-37+24}{12}=\frac{-13}{12}$$

(d) $\frac{6}{7}-\frac{2}{1}+\frac{-7}{9}+\frac{19}{21}=\frac{6}{7}-\frac{2}{1}-\frac{7}{9}+\frac{19}{21}$

$$=\frac{(6\times 9)-(2\times 63)-(7\times 7)+(19\times 3)}{63}$$

$$=\frac{54-126-49+57}{63}=\frac{-175+111}{63}=\frac{-64}{63}$$

Exercise 1C

1. Multiply :

(a) $\frac{3}{4}\times\frac{5}{7}=\frac{15}{28}$ (b) $\frac{3}{7}\times\frac{-4}{5}=\frac{-12}{35}$

(c) $\frac{-5}{9}\times\frac{4}{1}=\frac{-20}{9}$ (d) $\frac{-36}{7}\times\frac{-28}{9}=16$

2. Simplify :

(a) $\frac{-8}{7}\times\frac{14}{5}=\frac{-16}{5}$ (b) $\frac{13}{6}\times\frac{-18}{91}=\frac{-3}{7}$

(c) $\frac{-5}{9}\times\frac{72}{-125}=\frac{8}{25}$ (d) $\frac{-22}{9}\times\frac{-51}{-88}=\frac{-51}{36}$

(e) $\left(\frac{-16}{5}\times\frac{20}{8}\right)-\left(\frac{15}{5}\times\frac{-35}{3}\right)=-8+35=27$

(f) $\left(\frac{-3}{2}\times\frac{4}{5}\right)+\left(\frac{9}{5}\times\frac{-10}{3}\right)-\left(\frac{1}{2}\times\frac{3}{4}\right)$

$$\frac{-6}{5}+(-6)-\left(\frac{3}{8}\right)=\frac{-6}{5}-\frac{6}{1}-\frac{3}{8}$$

$$\frac{-6\times 8-6\times 40-3\times 5}{40}=\frac{-48-240-15}{40}=\frac{-303}{40}$$

3. Reciprocal of :

(a) $7=\frac{1}{7}$ (b) $-11=-\frac{1}{11}$

(c) $\frac{2}{5}=\frac{5}{2}$ (d) $\frac{-7}{15}=\frac{-15}{7}$

(e) $\frac{5}{12} = \frac{-12}{5}$

4. Reciprocal or multiplicative inverse of :

(a) $\frac{2}{5} \times \frac{4}{9} = \frac{8}{45} = \frac{45}{8}$

(b) $\frac{-3}{8} \times \frac{-7}{13} = \frac{21}{104} = \frac{104}{21}$

(c) $\frac{-5}{8} \times \frac{16}{15} = \frac{-2}{3} = \frac{-3}{2}$

(d) $\frac{-2}{1} \times \frac{-3}{5} = \frac{6}{5} = \frac{5}{6}$

5. $x = \frac{-3}{5}, y = \frac{7}{8}, z = \frac{-4}{9}$

To verify

(a) $x \times y = y \times x$

LHS $\frac{-3}{5} \times \frac{7}{8} = \frac{-21}{40}$

RHS $\frac{7}{8} \times \frac{-3}{5} = \frac{-21}{40}$

(b) $x \times (y \times z) = (x \times y) \times z$

LHS $\frac{-3}{5} \times \left(\frac{7}{8} \times \frac{-4}{9}\right) = \frac{-3}{5} \times \frac{-7}{18} = \frac{7}{30}$

RHS $(x \times y) \times z = \left(\frac{-3}{5} \times \frac{7}{8}\right) \times \frac{-4}{9} = \frac{-21}{40} \times \frac{-4}{9} = \frac{7}{30}$

(c) $x \times (y + z) = (x \times y) + (x \times z)$

LHS $\frac{-3}{5} \times \left(\frac{7}{8} - \frac{4}{9}\right) = \frac{-3}{5} \times \left(\frac{7 \times 9 - 4 \times 8}{72}\right)$

$= \frac{-3}{5} \times \left(\frac{63 - 32}{72}\right) = \frac{-3 \times 31}{5 \times 72} = \frac{-93}{360}$

RHS $x \times y + x \times z$

$$\left(\frac{-3}{5} \times \frac{7}{8}\right) + \left(\frac{-3}{5} \times \frac{-4}{9}\right) = \frac{-21}{40} + \frac{12}{45}$$

$$= \frac{-21 \times 9 + 12 \times 8}{360} = \frac{-189 + 96}{360} = \frac{-93}{360}$$

(d) $(x \times y)^{-1} = x^{-1} \times y^{-1}$

LHS $\left(\frac{-3}{5} \times \frac{7}{8}\right)^{-1} = \left(\frac{-21}{40}\right) = \frac{40}{-21}$

RHS $x^{-1} = y^{-1}$

$$\left(\frac{-3}{5}\right)^{-1} \times \left(\frac{7}{8}\right)^{-1} = \frac{5}{-3} \times \frac{8}{7} = \frac{40}{-21}$$

6. (a) $\frac{7}{4} \times \left(\frac{5}{8} + \frac{1}{3}\right) = \left(\frac{7}{4} \times \frac{5}{8}\right) + \left(\frac{7}{4} \times \frac{1}{3}\right) = \frac{35}{32} + \frac{7}{12}$

$$= \frac{35 \times 3 + 7 \times 8}{96} = \frac{105 + 56}{96} = \frac{161}{96}$$

(b) $\frac{-3}{8} \times \left(\frac{7}{16} - \frac{21}{4}\right) = \left(\frac{3}{8} \times \frac{7}{16}\right) - \left(\frac{-3}{8} \times \frac{21}{4}\right) = \frac{-21}{128} + \frac{63}{32}$

$$= \frac{-21 \times 1 + 63 \times 4}{128} = \frac{-21 + 252}{128} = \frac{231}{128}$$

(c) $\frac{3}{4} \times \left(\frac{8}{9} - \frac{30}{1}\right) = \left(\frac{3}{4} \times \frac{8}{9}\right) - \left(\frac{3}{4} \times \frac{30}{1}\right) = \frac{2}{3} - \frac{45}{2}$

$$= \frac{2 \times 2 - 45 \times 3}{6} = \frac{4 - 135}{6} = \frac{-131}{6}$$

7. Multiplicative inverse (reciprocol) of 6 :

(a) $5 = \frac{1}{5}$ (b) $-21 = \frac{1}{-21}$

(c) $\frac{15}{32} = \frac{32}{15}$ (d) $\frac{-17}{13} = \frac{13}{-17}$

(e) $\frac{11}{-5} = \frac{-5}{11}$ (f) $\frac{-7}{-8} = \frac{7}{8} = \frac{8}{7}$

(g) $-1 \times \frac{-5}{12} = \frac{5}{12} = \frac{12}{5}$ (h) $\frac{4}{5} \times \frac{15}{8} = \frac{3}{2} = \frac{2}{3}$

(i) $0 \times \frac{2}{3} = 0 = \infty$

8. (a) Existence of multiplicative inverse
   (b) Multiplication by 0
   (c) Existence of multiplicative identity
   (d) Existence of multiplicative inverse
   (e) Commulativity
   (f) Distributity of multiplication over addition
   (g) Distributivity of multiplication over addition

Exercise 1D

1. Divide

(a) $\frac{5}{9} \div \frac{25}{1} = \frac{5}{9} \times \frac{1}{25} = \frac{1}{45}$

(b) $\frac{7}{18} \div \frac{-28}{51} = \frac{7}{18} \times \frac{51}{-28} = \frac{-17}{24}$

(c) $\frac{20}{33} \div \frac{4}{-11} = \frac{20}{33} \times \frac{-11}{4} = \frac{-5}{3}$

(d) $\frac{-8}{9} \div \frac{4}{3} = \frac{-8}{9} \times \frac{3}{4} = \frac{-2}{3}$

(e) $\frac{-8}{13} \div \frac{3}{-26} = \frac{-8}{13} \times \frac{-26}{3} = \frac{16}{3}$

(f) $\frac{-24}{50} \div \frac{-4}{75} = \frac{24}{50} \times \frac{75}{4} = 9$

2. Evaluate

(a) $\left(\frac{5}{18} \div \frac{45}{72}\right) \div \left(\frac{-15}{18}\right) = \left(\frac{5}{18} \times \frac{72}{45}\right) \div \left(\frac{-15}{18}\right)$

$$\frac{4}{9} \div \left(\frac{-15}{18}\right) = \frac{4}{9} \times \frac{18}{-15} = \frac{8}{-15}$$

(b) $\left(\frac{-5}{23} \div \frac{15}{69}\right) \div \frac{-1}{5} = \left(\frac{-5}{23} \times \frac{69}{15}\right) \div \frac{-1}{5} = 1 \times \frac{5}{1} = 5$

3. Product of rational numbers $= -12$

   One number $= 8$

   Other number $= \quad -12 \div -8 = \frac{12}{8} = \frac{3}{2}$ or $1\frac{1}{2}$

4. $\frac{1}{17} \div \frac{-7}{85} = \frac{1}{17} \times \frac{85}{-7} = \frac{5}{-7}$ or $\frac{-5}{7}$

5. $24 \div \frac{-8}{13} = 24 \times \frac{13}{-8} = -39$

6. Sun of $\frac{3}{8}$ and $\frac{-5}{12}$

   $$\frac{3}{8} - \frac{5}{12} = \frac{3 \times 3 - 5 \times 2}{24} = \frac{9-10}{24} = \frac{-1}{24}$$

   Reciprocal of $\frac{-15}{8} \times \frac{16}{27} = \frac{-10}{9} = \frac{-9}{10}$

   $$\frac{-1}{24} \div \frac{-9}{10} = \frac{1}{24} \times \frac{10}{9} = \frac{5}{108}$$

7. Cost of $\frac{63}{5}$ kg mango is $=$ ₹ $\frac{1323}{4}$

   Cost of 1 kg mango is $= \frac{1323}{4} \times \frac{5}{63} = \frac{105}{4} =$ ₹ $26\frac{1}{4}$

8. In 1 hour the train covers $= 80\frac{4}{5}$ km or $\frac{404}{5}$ km

   In $4\frac{3}{4} = \frac{19}{4}$ hour the train covers $= \frac{404}{5} \times \frac{19}{4} = \frac{1919}{5} = 383\frac{4}{5}$ km

9. Area of rectangle $= 45\frac{5}{16} = \frac{725}{16}$ cm^2

Length = $\frac{25}{4}$ cm

Breadth = $\left(\frac{725}{16} \div \frac{25}{4}\right) = \frac{725}{16} \times \frac{4}{25} = \frac{29}{4} = 7\frac{1}{4}$ cm

Exercise 1E

1. Rational number between –2 and 6 is = $\frac{-2+6}{2} = \frac{4}{2} = 2$
2. Two rational numbers between –2 and 2 =

$$\frac{-2+2}{2} = 0 \text{ and } \frac{0-2}{2} = -1$$

So, numbers are –1, 0.

3. Rational number between $\frac{1}{3}$ and $\frac{1}{4}$

$$= \frac{1}{3} + \frac{1}{4} = \frac{4+3}{12} = \frac{7}{12} \times \frac{1}{2} = \frac{7}{24}$$

4. Three rational numbers between –2 and 5

$$\frac{2+5}{2} = \frac{3}{2}$$

$$\frac{-2}{1} + \frac{3}{2} = \frac{-4+3}{2} = \frac{-1}{2} \times \frac{1}{2} = \frac{-1}{4}$$

$$\frac{5}{1} + \frac{3}{2} = \frac{10+3}{2} = \frac{13}{2} \times \frac{1}{2} = \frac{13}{4}$$

So, numbers are $\frac{3}{2}, \frac{-1}{4}, \frac{13}{24}$

5. Four rational numbers between $\frac{1}{6}$ and $\frac{1}{3}$

$$\frac{1}{6} + \frac{1}{3} = \frac{1+2}{6} = \frac{3}{6} \times \frac{1}{2} = \frac{3}{12}$$

$$\frac{1}{6} + \frac{3}{12} = \frac{2+3}{12} = \frac{5}{12} \times \frac{1}{2} = \frac{5}{24}$$

$$\frac{3}{12} + \frac{1}{3} = \frac{3+4}{12} = \frac{7}{12} \times \frac{1}{2} = \frac{7}{24}$$

$$\frac{1}{6}+\frac{5}{24}=\frac{4+5}{24}=\frac{9}{24}\times\frac{1}{2}=\frac{9}{48}$$

So, numbers are; $\frac{3}{12},\frac{5}{24},\frac{7}{24}$ and $\frac{9}{48}$

7. Ten rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$

   By adopting the same procedure in sums 6 we can get the numbers as

   $$\frac{3}{8},\frac{5}{16},\frac{7}{16},\frac{9}{32},\frac{13}{32},\frac{15}{32},\frac{17}{64},\frac{19}{64},\frac{25}{64},\frac{27}{64}$$

8. Ten rational numbers between $\frac{2}{5}$ and $\frac{1}{2}$

   we have $\frac{1}{20},\frac{-7}{40},\frac{-23}{80},\frac{-5}{80},\frac{11}{40},\frac{13}{80},\frac{31}{80},\frac{53}{160},\frac{71}{180},\frac{115}{320}$

Check Your Mental Math IQ

A. Fill in the blanks :

1. $\frac{-4}{15}$	2. $\frac{-3}{4}$	3. $\frac{-5}{12}$	4. Negative
5. Negative	6. 1	7. No	8. –1
9. a	10. $\frac{-8}{17}$	11. $\frac{-5}{13}$	12. $\frac{1}{3}$
13. $\frac{3}{4},\frac{1}{2}$	14. $\frac{5}{7},\frac{-3}{4}$	15. 1	16. $\frac{-6}{13}$
17. $\frac{-11}{15}$	18. 1	19. $\frac{-7}{13}$	20. $\frac{-11}{23}$

B. 1. Zero 2. 1, –1 3. 0

C. 1. True 2. True 3. True 4. False 5. True

Multiple Choice Questions

1. (d)	2. (a)	3. (c)	4. (b)
5. (a)	6. (c)	7. (b)	8. (a)
9. (b)	10. (b)	11. (d)	

2 Exponents

Exercise 2A

1.

	Number	Base	Exponent
(a)	5^7	5	7
(b)	3^{-8}	3	–8
(c)	$(-5)^9$	–5	9
(d)	$\left(\sqrt{2}\right)^3$	$\sqrt{2}$	3
(e)	15	15	1
(f)	$\left(\frac{-4}{5}\right)^3$	$\frac{-4}{5}$	3
(g)	$\left(\frac{-3}{2}\right)^6$	$\frac{-3}{2}$	6
(h)	$\left(\sqrt{2}\right)^0$	$\sqrt{2}$	0

2. In exponential form :

(a) $8 \times 8 \times 8 \times 8 \times 8 = 8^5$

(b) $(-9) \times (-9) \times (-9) = (-9)^3$

(c) $ab \times ab \times ab \times ab \times ab = (ab)^5$

(d) $3^2 \times 3^2 \times 3^2 \times 3^2 = (3^2)^4 = 3^8$

(e) $\sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3} = \left(\sqrt{3}\right)^4$

(f) $\frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = \left(\frac{1}{5}\right)^3$

(g) $\frac{1}{8 \times 8 \times 8} = \left(\frac{1}{8}\right)^3$

(h) $\left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2} = \left(\frac{2}{5}\right)^{-6}$

3. In exponential form (Simplify)

(a) $9^{-3} \times 9^{-2} = 9^{-3-2} = 9^{-5}$ (b) $\frac{(3)^9}{(3)^5} = (3)^{9-5} = 3^4$

(c) $\{(-3)^5 \times 7^5\} = (-3 \times 7)^5 = (-21)^5$

(d) $\frac{x^{-3}}{x^{-5}} = (x)^{-3+5} = (x)^2$

(e) $\left(\frac{-3}{8}\right)^2 \times \left(\frac{-3}{8}\right)^5 \times \left(\frac{-3}{8}\right)^6 = \left(\frac{3}{8}\right)^{2+5+6} = \left(\frac{-3}{8}\right)^{13}$

(f) $(\sqrt{5})^3 \times (\sqrt{5})^{-1} \times \left(\sqrt{5}\right)^2 = (\sqrt{5})^{3-1+2} = \left(\sqrt{5}\right)^4$

(g) $(2^4)^{-3} = 2^{-12}$

4. Which is greater?

(a) 3^2 or 2^3

$$3^2 = 3 \times 3 = 9$$
$$2^3 = 2 \times 2 \times 2 = 8$$

So, $3^2 > 2^3$

(b) 3^4 or 4^3

$$3^4 = 3 \times 3 \times 3 \times 3 = 81$$
$$4^3 = 4 \times 4 \times 4 = 64$$

So, $3^4 > 4^3$

(c) 5^3 or 3^7

$$5^3 = 5 \times 5 \times 5 = 125$$
$$3^7 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 2187$$

So, $3^7 > 5^3$

5. (a) Expressing 625 as a power of 5

$$625 = 5 \times 5 \times 5 \times 5 = 5^4$$

(b) 343 as a power of 7

$$343 = 7 \times 7 \times 7 = 7^3$$

(c) 1024 as a power of –2

$$1024 = -2 \times -2 \times -2 \times -2 \times -2 \times -2 \times -2 \times -2 \times -2 \times -2$$

$$= (-2)^{10}$$

6. Reciprocol of

(a) $\frac{27}{64} = \frac{64}{27}$ or $\frac{4}{3} \times \frac{4}{3} \times \frac{4}{3} = \left(\frac{4}{3}\right)^3$

(b) $\frac{-16}{125} = \frac{125}{-16}$

(c) $\left(\frac{-3}{8}\right)^3 = \left(\frac{-8}{3}\right)^3$ or $\left(\frac{-3}{8}\right)^{-3}$

(d) $\left(\frac{-7}{3}\right)^2 = \left(\frac{3}{-7}\right)^2$ or $\left(\frac{-7}{3}\right)^{-2}$

7. (a) True (b) True (c) True (d) False
(e) False (f) True

8. Simplify :

(a) $\left(\frac{5}{7}\right)^3 \times \left(\frac{5}{7}\right)^{-5} = \left(\frac{5}{7}\right)^{3-5} = \left(\frac{5}{7}\right)^{-2} = \left(\frac{7}{5}\right)^2$

(b) $\left(\frac{-7}{9}\right)^8 \times \left(\frac{-7}{9}\right)^{-11} \times \left(\frac{-7}{9}\right)^1 = \left(\frac{-7}{9}\right)^{8-11+1} = \left(\frac{-7}{9}\right)^{-2} = \left(\frac{-9}{7}\right)^2$

(c) $\left(\frac{3}{11}\right)^4 \div \left(\frac{3}{11}\right)^{-3} = \left(\frac{3}{11}\right)^{4+3} = \left(\frac{3}{11}\right)^7$

(d) $\left(\frac{-5}{9}\right)^{-6} \div \left(\frac{-5}{9}\right)^5 = \left(\frac{-5}{9}\right)^{-6-5} = \left(\frac{-5}{9}\right)^{-11}$ or $\left(\frac{-9}{5}\right)^{11}$

(e) $\left(\frac{-4}{7}\right)^{-4} \div \left(\frac{-4}{7}\right)^{-10} = \left(\frac{-4}{7}\right)^{-4+10} = \left(\frac{-4}{7}\right)^6$

9. Simplify :

(a) $\left[\left(\frac{2}{5}\right)^{-3}\right]^4 = \left(\frac{2}{5}\right)^{-12} = \left(\frac{5}{2}\right)^{12}$

(b) $\left[\left(\frac{-6}{11}\right)^{-5}\right]^{-3} = \left(\frac{-6}{11}\right)^{15}$

(c) $\left(\frac{5}{7}\right)^{-1} \times \left(\frac{7}{3}\right)^{-1} = \frac{7}{5} \times \frac{3}{7} = \frac{3}{5}$

(d) $\left[4^{-1} \div 3^{-1}\right]^{-2} = \left[\frac{1}{4} \div \frac{1}{3}\right]^{-2} = \left[\frac{1}{4} \times \frac{3}{1}\right]^{-2}$

$\left[\frac{3}{4}\right]^{-2}$ or $\left[\frac{4}{3}\right]^{2}$ or $\frac{16}{9}$

(e) $\left(5^{-1} \times 3^{-1}\right) \div 6^{-1} = \left(\frac{1}{5} \times \frac{1}{3}\right) \div \frac{1}{6} = \frac{1}{15} \times \frac{6}{1} = \frac{2}{5}$

(f) $\frac{4^{-3} \times a^{-5} \times b^{-4}}{4^{-5} \times a^{-8} \times b^{3}} = 4^{-3+5} \times a^{-5+8} \times b^{-4-3} = 4^{2} \times a^{3} \times b^{-7} = \frac{16a^{3}}{b^{7}}$

(g) $(a^{3} \times a^{-2} \times a^{4})^{-2} = (a^{3} \times \frac{1}{a^{2}} \times a^{4})^{-2}$

$= (a^{3+4-2})^{-2} = (a^{5})^{-2} = a^{-10} = \frac{1}{a^{10}}$

10. Solve:

(a) $7^{x} = 343$

$7^{x} = 7 \times 7 \times 7 = 7^{3}$

So, $x = 3$

(b) $3^{3x} = \frac{1}{27} = \frac{1}{3 \times 3 \times 3} = \left(\frac{1}{3}\right)^{3}$ or 3^{-3}

$3x = -3$

$x = \frac{-3}{3} = -1$

(c) $\left(\sqrt{2}\right)^{x} = 2^{8} = \left(2^{1/2}\right)^{x} = 2^{8} = (2)^{x/2} = 2^{8}$

$\frac{x}{2} = 8 = x = 8 \times 2 = 16$

(d) $3^{2x+1} \div 9 = 27$

$3^{2x+1} \div 3^{2} = 27$

$$3^{2x+1-2} = 3^3$$
$$3^{2x-1} = 3^3$$
$$2x - 1 = 3$$
$$2x = 3 + 1 = 4$$
$$x = \frac{4}{2} = 2$$

(e) $5^{2x} \div 5^{-3} = 5^5$

$$5^{2x+3} = 5^5$$
$$2^{x+3} = 5$$
$$2x = 5 - 3 = 2$$
$$x = \frac{2}{2} = 1$$

11. $\left(\frac{5}{7}\right)^{-3} \times \left(\frac{5}{7}\right)^{-11} = \left(\frac{5}{7}\right)^{7x}$

$$\left(\frac{5}{7}\right)^{-3-11} = \left(\frac{5}{7}\right)^{7x}$$
$$\left(\frac{5}{7}\right)^{-14} = \left(\frac{5}{7}\right)^{7x}$$
$$-14 = 7x = x = \frac{-14}{7} = -2$$

12. $\left(\frac{2}{9}\right)^{3} \times \left(\frac{2}{9}\right)^{-6} = \left(\frac{2}{9}\right)^{2m-1}$

$$\left(\frac{2}{9}\right)^{3-6} = \left(\frac{2}{9}\right)^{2m-1}$$
$$\left(\frac{2}{9}\right)^{-3} = \left(\frac{2}{9}\right)^{2m-1}$$
$$2m - 1 = -3 \Rightarrow 2m = -3 + 1 \Rightarrow -2 = m = \frac{-2}{2} = -1$$

13. $$x = \left(\frac{3}{2}\right)^{2} \times \left(\frac{2}{3}\right)^{-4}$$

$$x = \left(\frac{3}{2}\right)^{2} \times \left(\frac{3}{2}\right)^{4}$$

$$x = \left(\frac{3}{2}\right)^{2+4}$$

$$x = \left(\frac{3}{2}\right)^{6}$$

$$x^{-2} = \left[\left(\frac{3}{2}\right)^{6}\right]^{-2}$$

$$x^{-2} = \left(\frac{3}{2}\right)^{-12} \text{ or } \left(\frac{2}{3}\right)^{12}$$

14. $$\left(\frac{5}{4}\right)^{-2} \div \left(\frac{1}{2}\right)^{-3} = \left(\frac{4}{5}\right)^{2} \div (2)^{3} = \left(\frac{4}{5} \times \frac{4}{5}\right) \div (8)$$

$$\frac{16}{25} \times \frac{1}{8} = \frac{2}{25}$$

Check Your Mental Maths

1. $5^{x} \div 5^{-3} = 5^{5}$

   $5^{x+3} = 5^{5}$

   $x + 3 = 5$

   $x = 5 - 3 = 2$

2. $$\left(\frac{-3}{4}\right)^{-2} = \left(\frac{4}{-3}\right)^{2} = \frac{4}{-3} \times \frac{4}{-3} = \frac{16}{9}$$

3. $$\frac{16}{81} = \frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3} = \left(\frac{2}{3}\right)^{4}$$

4. $$\left(\frac{x}{y}\right)^{0} = 1$$

5. $$(-5)^{-1} = \frac{-1}{5}$$

6. $$\left(\frac{5}{6}\right)^{7} \times \left(\frac{5}{6}\right)^{-9} = \left(\frac{5}{6}\right)^{2m}$$

$$\left(\frac{5}{6}\right)^{7-9} = \left(\frac{5}{6}\right)^{2m}$$

$$\left(\frac{5}{6}\right)^{-2} = \left(\frac{5}{6}\right)^{2m}$$

$$2m = -2$$

$$m = \frac{-2}{2} = -1$$

7. $(-3)^7 \div (-3)^2 = (-3)^{7-2} = (-3)^5$

8. $\left[\left(\frac{-3}{7}\right)^{-3}\right]^2 = \left(\frac{-3}{7}\right)^{-3\times 2} = \left(\frac{-3}{7}\right)^{-6}$ or $\left(\frac{-7}{3}\right)^6$

Multiple Choice Questions

1. $11^x = \frac{1}{121} = \left(\frac{1}{11}\right)^2$

   $11^x = (11)^{-2}$

   $x = -2$ (b)

2. $(1^3 + 2^3 + 3^3 + 4^3) = ?$

   $(1 + 8 + 27 + 64) = 100$ (d)

3. $27^{4/3} \div 27^{1/3} = ?$

   $27^{4/3 - 1/3} = 27^{\frac{4-1}{3}} = 27^{3/3} = 27^1 = 27$ (c)

4. $(x^3)^{-3} = x^{-9}$ (a)

5. $\left(\frac{-3}{7}\right)^3 = \frac{-3}{7} \times \frac{-3}{7} \times \frac{-3}{7} = \frac{-27}{343}$ (c)

6. $\left[\left(\frac{3}{5}\right)^0\right]^3 = \left(\frac{3}{5}\right)^0 = 1$ (c)

7. $\left(\frac{1}{3}\right)^0 + \left(\frac{1}{5}\right)^0 + \left(\frac{1}{7}\right)^0 = 1 + 1 + 1 = 3$ (a)

8. $\left(\frac{a}{b}\right)^0 \times \left(\frac{b}{c}\right)^0 \times \left(\frac{c}{a}\right)^0 = 1 \times 1 \times 1 = 1$ (b)

9. $3^{-1} \div 4^{-1} = \frac{1}{3} \div \frac{1}{4} = \frac{1}{3} \times \frac{4}{1} = \frac{4}{3}$ (a)

10. (c)

3 Square and Square Roots

Exercise 3A

1. Number which are perfect squares :

(a)

2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

$2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2 \times 2 \times 2 \times 2 = 16$

So, 256 is a perfect square.

(b)

5	625
5	125
5	25
	5

$5 \times 5 \times 5 \times 5$

$= 5 \times 5 = 25$

So, 625 is a perfect square.

(c)

2	576
2	288
2	144
2	72
2	36
2	18
3	9
	3

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$= 2 \times 2 \times 2 \times 3 = 24$

So, 516 is a perfect square.

(d)

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
	3

$2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$

$= 2 \times 2 \times 3 \times 3 = 36$

So, 1296 is a perfect square

(e)

2	2500
2	1250
5	625
5	125
5	25
	5

$2 \times 2 \times 5 \times 5 \times 5 \times 5$

$= 2 \times 5 \times 5 = 50$

So, 2500 is a perfect square

(f)

2	2116
2	1058
23	529
	23

$2 \times 2 \times 2 \times 23 \times 23$

$= 2 \times 23 = 46$

So, 2116 is a perfect square.

2. (a)

2	36
2	18
3	9
	3

$2 \times 2 \times 3 \times 3$

$= 2 \times 3 = 6$ is an even number

(b)

7	49
	7

$= 7 \times 7$

$=$ 7 is an odd number

(c)

2	324
2	162
3	81
3	27
3	9
	3

$2 \times 2 \times 3 \times 3 \times 3 \times 3$

$= 2 \times 3 \times 3 = 18$ is an even number

(d)

17	289
	17

17×17

$=$ 17 is an odd number

(e)

3	441
3	147
7	49
	7

$3 \times 3 \times 7 \times 7$

$= 3 \times 7 = 21$ is an odd number

(f)

3	1089
3	363
11	121
	11

$3 \times 3 \times 11 \times 11$

$= 3 \times 11 = 33$ is an odd number

3. (a)

2	1800
2	900
3	450
3	150
2	50
5	25
	5

$2 \times 2 \times 3 \times 3 \times 2 \times 5 \times 5$

To make it a perfect square it must be multiplied by 2

(b)

2	2904
2	1452
2	726
3	363
11	121
	11

$2 \times 2 \times 2 \times 3 \times 11 \times 11$

To make it a perfect square it must be multiplied by 6

(c)

3	1323
3	441
3	147
7	49
	7

$3 \times 3 \times 3 \times 7 \times 7$

So, it must be multiplied by 3

(d)

2	35280
2	17640
2	8820
3	4410
3	1470
7	490
7	70
2	10
	5

$2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7 \times 5$

So, to make it a perfect square it must be multiplied by 5

(e)

2	17640
2	8820
2	4410
3	2205
3	735
5	245
7	49
	7

$2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 \times 7$

So, to make it a perfect square it must be multiplied by $2 \times 5 = 10$

4. (a)

2	180
2	90
3	45
3	15
	5

$2 \times 2 \times 3 \times 3 \times 5$
To make it a perfect square it must be divided by 5

(b)

3	1575
3	525
5	175
5	35
	7

$3 \times 3 \times 5 \times 5 \times 7$
To make it a perfect square it must be divided by 7

(c)

2	3174
3	1587
23	529
	23

$2 \times 2 \times 3 \times 23 \times 23$
So, it must be divided by $2 \times 3 = 6$

(d)

2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$
= So, it must be divided by 3

(e)

2	16652
2	8326
	4163

$2 \times 2 \times 4163$
it must be divided by 4163

5. (a)

3	35943
	11981

It is not a perfect square because it does not make complete pairs.

(b)

2	57323
	28661

It does not make complete pairs.

(c)

2	333222
	166611

It does not make complete pairs.

(d)

3	23805
3	7935
5	2645
23	529
	23

$3 \times 3 \times 5 \times 23 \times 23$

It does not make complete pairs

6. Pythagorean triplets obey the following relation

$(2m)^2 + (m^2 - 1)^2 = (m^2 + 1)^2$

(a) For a given natural number '*m*' other than 1, we get a triplet such as $(2m), (m^2-1), (m^2 + 1)$

One member is 8 means $2m = 8$

So $m = \frac{8}{2} = 4$

$(2m), (m^2 - 1), (m^2 + 1)$

$(2\times4), (4^2 - 1), (4^2 + 1)$

8, 15, 17

$8^2 + 15^2 = 17^2$

$64 + 225 = 289$

So, other members of triplet are 15 and 17.

(b) Here $2m = 12$, $m = 6$

So $(m^2 - 1) = (6^2 - 1) = 35$

and $(m^2 + 1) = (6^2 + 1) = 37$

$12^2 + 35^2 = 37^2$

$144 + 1225 = 1369$ So, 35 and 37

(c) Here $2m = 10$, $m = 5$

So $(m^2 - 1) = 5^2 - 1 = 24$

and $m^2 + 1 = 5^2 + 1 = 26$

$(10)^2 + (24)^2 = (26)^2$

$100 + 576 = 676$ So, 24 and 26

(d) Here $2m = 14$, $m = 7$

So, $(m^2 - 1) = 7^2 - 1 = 48$

and $(m^2 + 1) = 7^2 + 1 = 50$

$(14)^2 + (48)^2 = (50)^2$

$196 + 2304 = 2500$

So, 48 and 50

(e) Here $2m = 16$; $m = 8$

So, $(m^2 - 1) = 8^2 - 1 = 63$

and $(m^2 + 1) = 8^2 + 1 = 65$

$(16)^2 + (63)^2 = (65)^2$

$256 + 3969 = 4225$

So, 63 and 65

7. (a) As we know,

Sum of first 5 odd numbers $= (5)^2 = 25$

So, $1 + 3 + 5 + 7 + 9 = 25$

(b) Sum of first 6 odd numbers $= (6)^2 = 36$

So, $1 + 3 + 5 + 7 + 9 + 11 = 36$

(c) Sum of first 7 odd numbers $= (7)^2 = 49$

So, $1+3+5+7+9+11+13 = 49$

(d) Sum of first 8 odd numbers $= (8)^2 = 64$

So, $1+3+5+7+9+11+13+15 = 64$

8. Given Pattern is :

$$1^2 = 1$$
$$11^2 = 121$$
$$111^2 = 12321$$

So (a) $(1111)^2 = 1234321$

(b) $111111^2 = 12345654321$

9. Given pattern is

$$11^2 = 121$$
$$101^2 = 10201$$
$$1001^2 = 1002001$$
$$10001^2 = 100020001$$
$$100001^2 = 10000200001$$

Exercise 3B

1. (a) Prime factorization of

2	144
2	72
2	36
2	18
3	9
3	3
	1

$2 \times 2 \times 2 \times 2 \times 3 \times 3$

Square root $= 2 \times 2 \times 3 = 12$

(b) 225 =

3	225
3	75
5	25
5	5
	1

$3 \times 3 \times 5 \times 5$

Square root $= 3 \times 5 = 15$

(c) 324 =

2	324
2	162
3	81
3	27
3	9
3	3
	1

$2 \times 2 \times 3 \times 3 \times 3 \times 3$

square root $= 2 \times 3 \times 3 = 18$

(d) 729 =

3	729
3	243
3	81
3	27
3	9
3	3
	1

$3 \times 3 \times 3 \times 3 \times 3 \times 3$

square root $= 3 \times 3 \times 3 = 27$

(e) 1296

Prime factors of 1296 $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$

Square root $= 2 \times 2 \times 3 \times 3 = 36$

(f) 2304

Prime factors of 2304 $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$

Square root $= 2 \times 2 \times 2 \times 2 \times 3 = 48$

(g) 7056

Prime factors of 7056 $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7$

Square root $= 2 \times 2 \times 3 \times 7 = 84$

(h) 11025

Prime factors of 11025 $= 3 \times 3 \times 5 \times 5 \times 7 \times 7$

Square root $= 3 \times 5 \times 7 = 105$

(i) 24336

Prime factors of 24336 $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 13 \times 13$

Square root $= 2 \times 2 \times 3 \times 13 = 156$

(j) 30625

Prime factors of 30625 $= 5 \times 5 \times 5 \times 5 \times 7 \times 7$

Square root $= 5 \times 5 \times 7 = 175$

2. Prime factors of 396 are

2	396
2	198
3	99
3	33
	11

$396 = 2 \times 2 \times 3 \times 3 \times 11$

To make it perfect square,
we should multiply it by 11

Now, square root $= 2 \times 3 \times 11 = 66$

3. Prime factors of 1152

2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
	3

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$

So, to make it a perfect square, we should divide it by 2

Square root $= 2 \times 2 \times 2 \times 3 = 24$

4. Square root of $1296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 = 36$

   So, number of rows $= 36$ and number of plants in each row $= 36$.

5. Square root of 202500

2	202500
2	101250
3	5625
3	1875
5	625
5	125
5	25
	5

$2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5$

$= 2 \times 3 \times 5 \times 5 = 150$

So, strength of the class $= 150$

6. First, we will take the LCM of 6, 8, 16

2	6, 8, 16
2	3, 4, 8
2	3, 2, 4
	3, 1, 2

$2 \times 2 \times 2 \times 3 \times 2 = 48$

To make it a perfect square we should multiply it by 3

So, least square number which is exactly divisible by each of

the numbers 6, 8, 16 = 48 × 3 = 144

7. LCM of 12, 15 and 20

2	12, 15, 20
2	6, 15, 10
3	3, 15, 5
5	1, 5, 5
	1, 5, 5

$2 \times 2 \times 3 \times 5$

To make it perfect square we should multiply it by $3 \times 5 = 15$

So, Least square number which is exactly divisible by each of the numbers 12, 15, and 20 is $2 \times 2 \times 3 \times 3 \times 5 \times 5 = 3600$

8. Total men = 335250

Left over men = 9

So, to arrange the other men in the form of square, we should take square root of 335250 – 9 = 335241

Prime factors of 335241 = 3 × 3 × 193 × 193

Square root = 3 × 193 = 579

So, the men in each row = 579

Exercise 3C

1. (a)

	234
2	54756
	4
43	147
	129
464	1856
	1856
	×

(b)

	625
6	390625
	36
122	306
	244
1245	6225
	6225
	×

(c)

```
        222
  2 | 4937284
    | 4
 42 |   93
    |   84
442  |    972
     |    884
4442 |     8884
     |     8884
     |       ×
```

(d)

```
       1213
  1 | 1471369
    | 1
 22 |   47
    |   44
241  |    313
     |    241
2423 |     7269
     |     7269
     |       ×
```

(e)

```
       3107
  3 | 9653449
    | 9
 61 |   65
    |   61
6207 |    43449
     |    43449
     |       ×
```

(f)

```
       4545
  4 | 20657025
    | 16
 85 |   465
    |   425
904  |    4070
     |    3616
9085 |     45425
     |     45425
     |       ×
```

(g)

```
        12351
  1 | 152547201
    | 1
 22 |   52
    |   44
243  |    854
     |    729
2465 |     12572
     |     12325
24701 |        24701
      |        24701
      |           ×
```

(h)

```
       9070
  9 | 82264900
    | 81
1807 |   12649
     |   12649
     |       ×
```

(i)

	56804
5	3226694416
	25
106	726
	636
1128	9069
	9024
113604	454416
	454416
	×

(j)

	7708
7	59413264
	49
147	1041
	1029
15408	123264
	123264
	×

2. **Number** **Number of digits in the square root**

(a) 441 $= \frac{3+1}{2} = 2$

(b) 67600 $= \frac{5+1}{2} = 3$

(c) 288369 $= \frac{6}{2} = 3$

3. (a)

	48
4	2361
	16
88	761
	704
	57

So, 57 should be subtracted.

(b)

	50
5	2509
	25
10	09
	00
	9

So, 9 should be subtracted.

(c)

	87
8	7581
	64
167	1181
	1169
	12

So, 12 should be subtracted

(d)

	208
2	43379
	4
408	3379
	3264
	95

So, 95 should be subtracted

(e)

```
        586
  5 | 343500
    | 25
108 |   935
    |   884
1166|    7100
    |    6996
    |     104
```

So, 104 should be subtracted.

4. (a)

```
        71
  7 | 4931
    | 49
141 |   31
    |   141
    | –110
```

So, 110 must be added.

(b)

```
        79
  7 | 6203
    | 49
149 |  1303
    |  1341
    |  –38
```

So, 38 must be added.

(c)

```
        712
  7 | 506900
    | 49
141 |   160
    |   141
1422|    2800
    |    2844
    |     –44
```

So, 44 should be added.

(d)

```
        443
  4 | 176201
    | 16
 84 |   362
    |   336
883 |    2601
    |    2649
    |     –48
```

So, 48 should be added.

(e)

```
        2125
  2 | 4515600
    | 4
 41 |   51
    |   41
422 |   1056
    |    844
4245|     21200
    |     21225
    |       –25
```

So, 25 should be added.

5. Largest 5-digit number is = 99999

```
      316
  3 | 99999
    | 9
 61 |   99
    |   61
626 |    3899
    |    3696
    |     203
```

So, 99999 – 203 = 99786 is the perfect square.

6. Least 6-digit number is 100000

```
      317
  3 | 100000
    | 9
 61 |   100
    |    61
621 |    3900
    |    4349
    |    –419
```

So, 100000 + 419 = 100419 is the perfect square.

7.

```
      89
  8 | 7912
    | 64
169 | 1512
    | 1521
    |  –9
```

So, 7912 + 9 = 7921 is the perfect square and 79 will be the square root.

8.

```
      208
  2 | 43379
    | 4
408 |   3379
    |   3264
    |    115
```

So, 43379 – 115 = 43264 is the perfect square and 208 is the square root.

9. First we must subtract 10 from 120419 cause the general need 10 extra men.

So, 120419 – 10 = 120409

```
      347
  3 | 120409
    | 9
 64 |   304
    |   256
687 |    4809
    |    4809
    |      ×
```

So, in front row 347 men were there.

10. Side of the square field is = $\sqrt{60025}$

```
      245
  2 | 60025
    | 4
 44 |   200
    |   176
485 |    2425
    |    2425
    |      ×
```

Side = 245m

Perimeter = 4 × 245 = 980m

Speed = 18km/hr *or*

= 5 m/sec

So, to cover 980 m the time he will take $\frac{980}{5} = 196$ sec

Exercise 3D

1. (a) $\sqrt{\frac{529}{841}} = \sqrt{\frac{529}{841}} = \sqrt{\frac{23\times23}{29\times29}} = \frac{23}{29}$

(b) $\sqrt{\frac{81}{225}} = \sqrt{\frac{81}{225}} = \sqrt{\frac{9\times9}{15\times15}} = \frac{9}{15}$

(c) $\sqrt{2\frac{7}{9}} = \sqrt{\frac{25}{9}} = \sqrt{\frac{5\times5}{3\times3}} = \frac{5}{3}$ or $1\frac{2}{3}$

(d) $\sqrt{1\frac{56}{169}} = \sqrt{\frac{225}{169}} = \sqrt{\frac{15\times15}{13\times13}} = \frac{15}{13}$ or $1\frac{2}{13}$

(e) $\sqrt{6\frac{115}{289}} = \sqrt{\frac{1849}{289}} = \sqrt{\frac{43\times43}{17\times17}} = \frac{43}{17}$ or $2\frac{9}{17}$

(f) $25\frac{544}{729} = \sqrt{\frac{18769}{225}} = \sqrt{\frac{137\times137}{27\times27}} = \frac{137}{27}$

```
      137                          27
    1 | 18769                  2 | 729
      | 1                        | 4
   23 |  87                 47 |  329
      |  69                      |  329
  267 |   1869                   |    ×
      |   1869
      |    ×
```

(g) $\sqrt{3\frac{234}{3025}} = \sqrt{\frac{9409}{3025}} = \sqrt{\frac{97\times97}{55\times55}} = \frac{97}{55}$

```
      97                           55
    9 | 9409                   5 | 3025
      | 81                       | 25
  187 |  1309                105 |  525
      |  1309                    |  525
      |    ×                     |    ×
```

(h) $\sqrt{42\frac{583}{1369}} = \sqrt{\frac{58081}{1369}} = \sqrt{\frac{241\times241}{37\times37}} = \frac{241}{37}$

```
      241                          37
    2 | 58081                  3 | 1369
      | 4                        | 9
   44 |  180                 67 |  469
      |  176                     |  469
  401 |   481                    |    ×
      |   481
      |    ×
```

2. (a) $\sqrt{\frac{80}{405}} = \sqrt{\frac{16}{81}} = \sqrt{\frac{4\times4}{9\times9}} = \frac{4}{9}$

(b) $\sqrt{\frac{243}{867}} = \sqrt{\frac{81}{289}} = \sqrt{\frac{9\times9}{17\times17}} = \frac{9}{17}$

(c) $\sqrt{\frac{1.44}{2.25}} = \sqrt{\frac{144}{225}} = \sqrt{\frac{12\times12}{15\times15}} = \frac{12}{15}$

(d) $\sqrt{\dfrac{36.1}{102.4}} = \sqrt{\dfrac{361}{1024}} = \sqrt{\dfrac{19\times19}{32\times32}} = \dfrac{19}{32}$

3. (a)

```
       1.11
  1 | 1.2321
    | 1
 21 |   23
    |   21
221  |    221
     |    221
     |      ×
```

(b)

```
       2.22
  2 | 4.9284
    | 4
 42 |   92
    |   84
442  |    884
     |    884
     |      ×
```

(c)

```
       3.33
  3 | 11.0889
    | 9
 63 |   208
    |   189
663  |    1989
     |    1989
     |      ×
```

(d)

```
       0.85
  8 | 0.7225
    |   64
165  |    825
     |    825
     |      ×
```

(e)

```
       0.222
  2 | 0.049284
    |   4
 42 |     92
    |     84
442  |      884
     |      884
     |        ×
```

(f)

```
       0.333
  3 | 0.110889
    |   9
 63 |    208
    |    189
663  |     1989
     |     1989
     |       ×
```

(g)

```
        3.767
   3 | 14.190289
     |  9
  67 |   519
     |   469
 746 |     5002
     |     4476
7527 |       52689
     |       52689
     |         ×
```

(h)

```
        12.35
   1 | 152.5225
     |  1
  22 |   52
     |   44
 243 |    852
     |    729
2465 |     12325
     |     12325
     |        ×
```

4. $\sqrt{\frac{3}{7}} = 0.42857142$ (after dividing 3 by 7)

```
        0.6546
   6 | 0.42857142
     |  36
 125 |   685
     |   625
1304 |    6071
     |    5216
13086 |     85542
      |     78516
      |      7026
```

So, 0.6546

5.

```
        125
   1 | 15625
     | 1
  22 |  56
     |  44
 245 |   1225
     |   1225
     |      ×
```

$\therefore$ 15625 = 125

So $\sqrt{156.25} = 12.5$

and $\sqrt{1.5625} = 1.25$

and $\sqrt{156.25} + \sqrt{1.5625}$

12.50 + 1.25 = 13.75

6. (a)

	1.732
1	3.000000
	1
27	200
	189
343	1100
	1029
3462	7100
	6924
	176

Ans. 1.732

(b)

	2.236
2	5.000000
	4
42	100
	84
443	1600
	1329
4466	27100
	26796
	304

Ans. : 2.236

(c)

	3.162
3	10.000000
	9
61	100
	61
626	3900
	3756
6322	14400
	12644
	1756

So, 30162

(d)

	15.414
1	237.615000
	1
25	137
	125
304	1261
	1216
3081	4550
	3081
30821	146900
	123284
	23616

Ans. : 15.414

(e) $10\frac{2}{3} = \frac{32}{3} = 10.666667$

	3.265
3	10.666667
	9
62	166
	124
646	4266
	3876
6525	39067
	32625
	6442

So, 3.265

7. $\sqrt{103.0225}$

```
        10.15
   1 | 103.0225
     | 1
 201 |   0302
     |    201
2025 |    10125
     |    10125
     |       ×
```

$\because \sqrt{103.0225} = 10.15$

$so, \sqrt{10302.25} = 101.5$

$and \sqrt{1.030225} = 1.015$

8. Area of the square field = $325m^2$
   Side of the square field = $\sqrt{325}$

```
        18.027
   1 | 325.000000
     | 1
  28 |  225
     |  224
3602 |     10000
     |     7204
36047 |      279600
     |      252329
     |       27271
```

So, side = 18.027m

Check Your Mental Maths IQ

1. 63 = 3 × 3 × 7
   To make 63 a perfect square we should divide it by 7

2. $\sqrt{20\frac{1}{4}} = \sqrt{\frac{81}{4}} = \sqrt{\frac{9\times 9}{2\times 2}} = \frac{9}{2}$ or $4\frac{1}{2}$

3.

	0.447
4	0.20000
	16
84	400
	336
887	6400
	6209
	191

4. $(50)^2 - (49)^2$

$= (50 + 49)(50 - 49)$

$= 99 \times 1 = 99$

5. No

6.

	8
8	79
	64
	15

and

	9
9	79
	81
	–2

So, 2 must be added to make it perfect rsquare.

7. $\sqrt{0.81} = \sqrt{\frac{81}{100}} = \sqrt{\frac{9 \times 9}{10 \times 10}} = \frac{9}{10} = 0.9$

Multiple Choice Questions

1. (c) $63 = 3 \times 3 \times 7$

   To make it perfect square it must be divided by 7

2. (c) $\sqrt{5^2 \times 4^2 \times 36} = ?$

   $\sqrt{5^2 \times 4^2 \times 6 \times 6} = \sqrt{5^2 \times 4^2 \times 6^2} = 5 \times 4 \times 6 = 120$

3. (d) We know sum of first 10 odd numbers $= (10)^2 = 100$

4. (b)

	15
1	250
	1
25	150
	125
	25

So, 25 must be added to make it a perfect square

5. (a) $25^2 - 24^2 = (25 + 24)(25 - 24) = 49 \times 1 = 49$

6. (d)

$$\begin{array}{r|l} & 25 \\ \hline 2 & 600 \\ & 4 \\ \hline 45 & 200 \\ & 225 \\ \hline & 25 \\ \hline \end{array}$$

So, 25 must be added to make it a perfect square

7. (b) 121; $\therefore$ Unit place is odd
8. (b) Smallest three-digit number which is a perfect square is 100
9. (c) Value of

$$\sqrt{49}+\sqrt{0.49}+\sqrt{0.0049}=7+0.7+0.07=7.77$$

4 Cube and Cube Roots

Exercise 4A

1. (a) $(7)^3=7\times7\times7=343$

   (b) $(12)^3=12\times12\times12=1728$

   (c) $(25)^3=25\times25\times25=15625$

   (d) $(30)^3=30\times30\times30=27000$

   (e) $(45)^3=45\times45\times45=91125$

   (f) $\left(\frac{3}{5}\right)^3=\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}=\frac{27}{125}$

   (g) $\left(\frac{-4}{9}\right)^3=\frac{-4}{9}\times\frac{-4}{9}\times\frac{-4}{9}=\frac{-64}{729}$

   (h) $\left(2\frac{5}{7}\right)^3=\left(\frac{19}{7}\right)^3=\frac{19}{7}\times\frac{19}{7}\times\frac{19}{7}=\frac{6859}{343}=19\frac{342}{343}$

   (i) $\left(1\frac{9}{10}\right)^3=\left(\frac{19}{10}\right)^3=\frac{19}{10}\times\frac{19}{10}\times\frac{19}{10}=\frac{6859}{1000}=6\frac{859}{1000}$

(j) $(0.03)^3 = \frac{3}{100} \times \frac{3}{100} \times \frac{3}{100} = \frac{27}{1000000} = 0.000027$

2. (a)

2	108
2	54
3	27
3	9
3	3
	1

No

(b)

7	343
7	49
	7

Yes

(c)

2	612
2	306
3	153
3	51
	17

No

(d)

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
	3

Yes

(e)

2	2744
2	1372
2	686
7	343
7	49
	7

Yes

(f)

2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
	5

No

(g)

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

Yes

(h)

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

Yes

(i)

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

Yes

3. 64, 512, 1000, 1728 and 13824
4. 125, 343, 1331, 3375 and 6859
5.

3	675
3	225
3	75
5	25
5	5
	1

$3 \times 3 \times 3 \times 5 \times 5$

So, it should be multiplied by 5 to make a perfect cube

Required perfect cube $= 3 \times 5 = 15$

6.

2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

It must be divided by $2 \times 2 = 4$

So perfect cube $= 3 \times 3 = 9$

7. Volume of the cube $= (\text{side})^3 = (3.8)^3$

$= 3.8 \times 3.8 \times 3.8 = 54.872$

Exercise 4B

1. (a)

2	216
2	108
2	54
3	27
3	9
3	3
	1

$2 \times 2 \times 2 \times 3 \times 3 \times 3$

$2 \times 3 = 6$

(b) $343 = 7 \times 7 \times 7$; So cube root $= 7$

(c)

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$= 2 \times 2 \times 3 = 12$

(d)

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$2 \times 2 \times 2 \times 7 \times 7 \times 7$

$= 2 \times 7 = 14$

(e)

2	10648
2	5324
2	2662
11	1331
11	121
11	11
	1

$2 \times 2 \times 2 \times 11 \times 11 \times 11$

$= 2 \times 11 = 22$

(f)

2	27000
2	13500
2	6750
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

$2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$

$= 2 \times 3 \times 5 = 30$

(g)

3	91125
3	30375
3	10125
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$

$= 3 \times 3 \times 5 = 45$

(h)

17	4913
17	289
17	17
	1

$17 \times 17 \times 17 = 17$ (put – sign)

So cube root = –7

(i)

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$= 2 \times 3 \times 3 = 18$ (put –ve sign)

$= -18$

(j) –32768

2	32768
2	16384
2	8192
2	4016
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times$
$2 \times 2 \times 2 \times 2 \times 2$

$= 2 \times 2 \times 2 \times 2 \times 2 = 32$ (put –ve sign)

-32

2. Show that

(a) $\sqrt[3]{125 \times 64} = \sqrt[3]{125} \times \sqrt[3]{64}$

LHS $\sqrt[3]{125 \times 64} = \sqrt[3]{5 \times 5 \times 5 \times 4 \times 4 \times 4} = 5 \times 4 = 20$

RHS $\sqrt[3]{125} = \sqrt[3]{64} = 5 \times 4 = 20$

So, LHS = RHS (Hence proved)

(b) $\sqrt[3]{216\times(-343)} = \sqrt[3]{216}\times\sqrt[3]{-343}$

LHS $\sqrt[3]{6\times6\times6\times(-7)\times(-7)\times(-7)} = 6\times-7 = -42$

RHS $\sqrt[3]{216}\times\sqrt[3]{-343} = 6\times-7 = -42$

LHS = RHS (Hence proved)

(c) $\sqrt[3]{(-216)\times1728} = \sqrt[3]{-216}\times\sqrt[3]{1728}$

LHS $\sqrt[3]{(-216)\times1728} = -6\times12 = -72$

RHS $\sqrt[3]{-216}\times\sqrt[3]{1728} = -6\times12 = -72$

LHS = RHS (Hence proved)

3. Evaluate

(a) $\sqrt[3]{1372}\times\sqrt[3]{1458}$

2	1372
2	686
7	343
7	49
	7

2	1458
3	729
3	243
3	81
3	27
3	9
	3

$\sqrt[3]{1372}\times\sqrt[3]{1458}$

$2\times2\times7\times7\times7\times2\times3\times3$
$\times3\times3\times3\times3$

$= 2\times7\times3\times3 = 126$

(b) $\sqrt[3]{392}\times\sqrt[3]{448}$

2	392
2	196
2	98
7	49
7	7
	1

2	448
2	224
2	112
2	56
2	28
2	14
7	7
	1

$\sqrt[3]{392} = 2\times2\times2\times7\times7$

$\sqrt[3]{448} = 2\times2\times2\times2\times2\times2\times7$

So, $2\times2\times2\times7 = 56$

(c) $\sqrt[3]{3375 \times 729}$

3	3375
3	1125
3	375
5	125
5	25
5	5
	1

3	729
3	243
3	81
3	27
3	9
3	3
	1

$3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 3 \times 3 \times 3 \times 3 \times 3$
$= 3 \times 5 \times 3 \times 3 = 135$

4. (a) $\sqrt[3]{\frac{1331}{4096}} = \sqrt[3]{\frac{11 \times 11 \times 11}{16 \times 16 \times 16}} = \frac{11}{16}$

(b) $\sqrt[3]{\frac{-2197}{9261}} = \sqrt[3]{\frac{-13 \times -13 \times -13}{21 \times 21 \times 21}} = \frac{-13}{21}$

(c) $\sqrt[3]{\frac{4096}{-2197}} = \sqrt[3]{\frac{16 \times 16 \times 16}{-13 \times -13 \times -13}} = \frac{16}{-13}$

(d) $\sqrt[3]{\frac{-3375}{-2744}} = \sqrt[3]{\frac{15 \times 15 \times 15}{14 \times 14 \times 14}} = \frac{15}{14}$

5. $\sqrt[3]{0.001331} = \sqrt[3]{\frac{1331}{1000000}} = \sqrt{\frac{11 \times 11 \times 11}{100 \times 100 \times 100}} = \frac{11}{10} = 0.11$

6.

2	17496
2	8748
2	4374
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

$2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

So, to make it a perfect we must multiply it by $3 \times 3 = 9$

So, cube root =

$2 \times 3 \times 3 \times 3 = 54$

7.

3	1323
3	441
3	147
7	49
7	7
	1

$3 \times 3 \times 3 \times 7 \times 7$

To make it a perfect cube it must be divided by $7 \times 7 = 49$

So, cube root of the number = 3

8. Volume of the cubical box = 13.824 cubic metres

Length of each side of the box = $\sqrt[3]{13.824} = \sqrt[3]{\frac{13824}{1000}}$

$$\sqrt[3]{\frac{13824}{1000}} = \sqrt{\frac{24 \times 24 \times 24}{10 \times 10 \times 10}} = \frac{24}{10} = 2.4 \text{ m}$$

Check Your Mental Math IQ

A. State True or False :

1. True 2. True 3. True 4. False
5. False

B. 1. No 2. 7 3. 2 4. 0.4
5. 6/7 6. –50 7. $4 \times 6 = 24$

Multiple Choice Questions

1. (a) 2. (c) 3. (b) 4. (b)
5. (a) 6. (a) 7. (c) 8. (b)
9. (b) 10. (b)

5 Algebra

Algebraic Expressions Special Products

Exercise 5A

1. Which of the following expressions are polynomials?

   Polynomials : An algebraic expression in which the variables involved have only non-negative integral powers is called a **polynomial**.

   (a) So, for $5x^2 + 10x - 4 =$ Yes

   (b) $3x^{1/2} - 5x + 1 =$ No

   (c) $\frac{2}{3}x^3 + 7x^2 - 16$ = Yes

   (d) $6x^3 - 3x^2 + 6\sqrt{x} + 2$ No

   (e) $7x^2y^2 + 3xy^2 + 5x^2y - 7x$ = Yes

   (f) $4x^2y^{1/2} - 3xy + 3x^{3/2}y =$ No

2. (a) $4y - 5 + \frac{1}{2}y^2$

   Degree is highest power of the variables, so degree = 2

   (b) $5x^4 - 7x^2 + \frac{5}{9}$ Degree = 4

   (c) $5x^3 - 2x^2y^2 + x^2 + 9y^2$; Degree = 4

   (d) $5x^2y^3 - 3xy^2 + 6 - \sqrt{2}\,x^4$; Degree = 5

   (e) $5x^5 - 7x^5y + 4xy^4 + 7y^8$

   Sum of powers of first term = 5

   Sum of powers of second term = 5 + 1 = 6

   Sum of powers of third term = 4

   Sum of powers of fourth term = 8

   So, degree of the polyonomial = 8

3. (a) $5x^3 + 7x^4 - 8x + 3x^2 - 3$

   In ascending order it is written as

$-3-8x+3x^2+5x^3+7x^4$

(b) $6+x^2-18x+36x^3$

In ascending order,

$6-18x+x^2+36x^3$

(c) $\frac{1}{6}x^4-x^5+2x-5x^3-2$

In ascending order $-2+2x-5x^3+\frac{1}{6}x^4-x^5$

4. (a) $-15x^8+3x^4y+5xy^6+2x^3y^3$

In descending order,

$-15x^8+5xy^6+2x^3y^3+3x^4y$

(b) $-2x+3+5x^2-3x^4-x^5$

In descending order

$-x^5-3x^4+5x^2-2x+3$

(c) $5xy^2-2x^3y-15x^2+3x^4y$

In descending order,

$3x^4y-2x^3y+5xy^2-15x^2$

5. Add the following polynomials :

(a)
$$\begin{array}{l} 7a-3b+5c \\ 2a-3b-4c \\ +4a+b+c \\ \hline 13a-5b+2c \\ \hline \end{array}$$

(b)
$$\begin{array}{l} -7x^2-3xy+10y^2 \\ 2x^2+8xy-11y^2 \\ -3x^2+6xy+8y^2 \\ \hline -8x^2+11xy+7y^2 \\ \hline \end{array}$$

(c)
$$\begin{array}{l} 3x^2-3xy+5y^2 \\ 7x^2+4xy-2y^2 \\ 5x^2 \qquad\quad +1y^2 \\ \hline 15x^2+1xy+4y^2 \\ \hline \end{array}$$

(d)
$$\begin{array}{l} 5x^2-7xy+4y^2-3x \\ 4x^2+2xy-1y^2 \qquad\quad +1y \\ 1x^2+5xy-2y^2+3x-1y \\ \hline 10x^2+ \qquad +1y^2 \\ \hline \end{array}$$

6. Subtract

(a)
$$\begin{array}{l} 25x^2+16xy-3b^2-2 \\ -6x^2+13xy \qquad\quad -1+4a^2 \\ + \quad - \qquad\qquad\qquad + \quad - \\ \hline 31x^2+3xy-3b^2-1+4a^2 \\ \hline \end{array}$$

(b) $-x^2+2xy^2-3x^2y+y^3$

$x^3-1xy^2+5x^2y-y^3$

$-\quad +\quad -\quad +$

$-2x^3+3xy^2-8x^2y+2y^3$

(c) $\frac{4}{3}x^2y+5x^3-\frac{2}{3}y^3+5xy^2$

$3x^2y+4x^3+5y^3-\frac{1}{2}xy^2$

$-\quad -\quad -\quad +$

$\frac{5}{3}x^2y+1x^3-\frac{17}{3}y^3+\frac{11}{2}xy^2$

(d) $\frac{1}{9}x^3-\frac{3}{7}x^2+\frac{2}{5}x+5$

$\frac{-8}{+9}x^3+\frac{4}{-7}x^2-\frac{3}{+5}x+1$

$-$

x^3-1x^2+1x+4

7. $2x^3-8x^2+9x-10$

$\mp 7x^3 \quad \pm 8x \mp 9$

$9x^3-8x^2+1x-1$

So, $9x^3-8x^2+1x-1$ must be added

8. $x^4-6x^3+x^2-3x+1$

$x^5-7x^3+x^2-6x+8$

$-\quad +\quad -\quad +\quad -$

$-x^5+x^4+x^3 \quad +3x-7$

9. Multiply :

(a) $3x^3 \times 4x^4 = 12x^7$

(b) $4a^2b \times -6a^3b^2c = -24a^5b^4c$

(c) $-6/8x^4yz \times 24x^2y^2z^3 = -18x^6y^3z^4$

(d) $-5/8x^2y^3 \times -16/15x^3y = 2/3x^5y^4$

10. (a)

$$\begin{array}{r} 3x^2 - 2x + 5 \\ \times\ x - 3 \\ \hline -9x^2 + 6x - 15 \\ 3x^3 - 2x^2 + 5x \\ \hline 3x^3 - 11x^2 + 11x - 15 \\ \hline \end{array}$$

(b)

$$\begin{array}{r} x^2 + 2x + 1 \\ \times\ 2x + 3 \\ \hline 3x^2 + 6x + 3 \\ 2x^2 + 4x^2 + 2x \\ \hline 2x^3 + 7x^2 + 8x + 3 \\ \hline \end{array}$$

(c)

$$\begin{array}{r} 2x^2 + x - 5 \\ \times\ x^2 - 2x + 3 \\ \hline 6x^2 + 3x - 15 \\ 4x^3 - 2x^2 + 10x \\ 2x^4 + 1x^3 - 5x^2 \\ \hline 2x^4 + 5x^3 - 1x^2 + 13x - 15 \\ \hline \end{array}$$

(d)

$$\begin{array}{r} 3x^2 - 5x + 6 \\ \times\ 3 - 5x^2 \\ \hline -15x^4 + 25x^3 - 30x^2 \\ +9x^2 - 15x + 18 \\ \hline -15x^4 + 25x^3 - 21x^2 - 15x + 18 \\ \hline \end{array}$$

(e)

$$\begin{array}{rrrrr} & & & 3x^5 - 7x^3 + 2x^2 - x + 4 & \\ & & & \times\ x^3 - 2x^2 + 3x - 1 & \\ \hline & & & -3x^5 + 7x^3 - 2x^2 + x - 4 & \\ & 9x^6 & +6x^3 - 3x^2 + 12x & & -21x^4 \\ & -6x^7 & +14x^5 - 2x^3 - 8x^2 & & -x^4 \\ & 8x^8 - 7x^6 + 2x^5 + 4x & & & -4x^4 \\ \hline 3x^8 - 6x^7 + 2x^6 + 13x^5 + 15x^3 - 13x^2 + 13x - 4 - 26x^4 & & & & \\ \hline \end{array}$$

(f)

$$\begin{array}{r} 5x^2 - 7x + 2 \\ \times\ 2x^2 - 3x - 5 \\ \hline -25x^2 + 35x - 10 \\ -15x^3 + 21x^2 - 6x \\ 10x^4 - 14x^3 + 4x^2 \\ \hline 10x^4 - 29x^3 + 29x - 10 \\ \hline \end{array}$$

(g)

$$\begin{array}{r} (5x - 7) \\ \times\ (2x + 3) \\ \hline 15x - 21 \\ 10x^2 - 14x \\ \hline 10x^2 + 1x - 21 \\ \hline \end{array}$$

$$\begin{array}{r} (10x^2 + x - 21) \\ \times\ (7x - 8) \\ \hline -80x^2 - 8x + 168 \\ 70x^3 + 7x^2 - 147x \\ \hline 70x^3 - 73x^2 - 155x + 168 \\ \hline \end{array}$$

Exercise 5B

1. Divide

(a) $x^7 \div x^3 = x^{7-3} = x^4$

(b) $-56abc \div 7b = -8ac$

(c) $\dfrac{36xy^2z}{-4xy} = -9yz$

(d) $-75x^2y^5z \div 15xy^2z \quad = 5x^{2-1}y^{5-2}z^{1-1}$

$= 5xy^3z^0 = 5xy^3$

(e) $\dfrac{-63x^2y^3z^4}{9x^4y^2z^7} = -7x^{2-4}y^{3-2}z^{4-7}$

$= -7x^{-2}yz^{-3}$

$= \dfrac{-7y}{x^2z^3}$

2. (a)

$$
\begin{array}{r|l}
 & -2x^2+8x-4 \\
\hline
-4x^2 & 8x^4-32x^3+16x^2 \\
 & -8x^4 \\
\hline
 & -32x^3 \\
 & \pm 32x^3 \\
\hline
 & 16x^2 \\
 & -16x^2 \\
\hline
 & \times
\end{array}
$$

Ans. : $-2x^2+8x-4$

(b) $(8a^2b^2 - 6ab^2 + 10a^2b^3)$ by $2ab$

First in descending order of its degree first, we will write the polynomial as $10a^2b^3 + 8a^2b^2 - 6ab^2$

$$
\begin{array}{r|l}
 & 5ab^2+4ab-3b \\
\hline
2ab & 10a^2b^3+8a^2b^2-6ab^2 \\
 & -10a^2b^3 \\
\hline
 & 8a^2b^2 \\
 & -8a^2b^2 \\
\hline
 & -6ab^2 \\
 & -6ab^2 \\
 & + \\
\hline
 & \times
\end{array}
$$

Ans. : $5ab^2+4ab-3b$

(c)

$$
\begin{array}{r|l}
 & -8x^2-16x+2 \\
\hline
-\frac{1}{2}x & 4x^2+8x^2-x \\
 & -4x^2 \\
\hline
 & 8x^2 \\
 & -8x^2 \\
\hline
 & -x \\
 & -x \\
 & + \\
\hline
 & \times \\
\hline
\end{array}
$$

Ans. : $-8x^2-16x+2$

(d)

$$
\begin{array}{r|l}
 & 2x^3+6x^2-1 \\
\hline
3x^2 & 6x^5+18x^4-3x^2 \\
 & -6x^5 \\
\hline
 & 18x^4 \\
 & -18x^4 \\
\hline
 & -3x^2 \\
 & -3x^2 \\
 & + \\
\hline
 & \times \\
\hline
\end{array}
$$

Ans. : $2x^3+6x^2-1$

(e)

$$
\begin{array}{r|l}
 & 10x^3+6xy-5 \\
\hline
2xy & 20x^3y+12x^2y^2-10xy \\
 & -20x^3y \\
\hline
 & 12x^2y^2 \\
 & -12x^2y^2 \\
\hline
 & -10xy \\
 & -10xy \\
 & + \\
\hline
 & \times \\
\hline
\end{array}
$$

Ans. : $10x^2+6xy-5$

(f)

$$
\begin{array}{r|l}
 & -18ab+9c-27d \\
\hline
-\frac{1}{3}ab & 6a^2b^2-3abc+9abd \\
 & -6a^2b^2 \\
\hline
 & -3abc \\
 & -3abc \\
 & + \\
\hline
 & 9abd \\
 & 9abd \\
 & - \\
\hline
 & \times \\
\hline
\end{array}
$$

Ans. : $-18ab+9c-27d$

3. (a) $2x^2 - 11x + 12$ by $x - 4$

$$
\begin{array}{r|l}
 & 2x - 3 \\
\hline
x-4 & 2x^2 - 11x + 12 \\
 & \underline{\underset{-}{2x^2} \underset{+}{-} 8x} \\
 & \quad -3x + 12 \\
 & \quad \underline{\underset{+}{-3x} \underset{-}{+} 12} \\
 & \qquad \times
\end{array}
$$

Ans. : $2x - 3$

(b)

$$
\begin{array}{r|l}
 & -2x + 7 \\
\hline
3x-4 & 29x - 6x^2 - 28 \\
 & \underline{\underset{-}{8x} \underset{+}{-} 6x^2} \\
 & \quad 21x - 28 \\
 & \quad \underline{\underset{-}{21x} \underset{+}{-} 28} \\
 & \qquad \times
\end{array}
$$

Ans. : $-2x + 7$

(c)

$$
\begin{array}{r|l}
 & 5x^2 - 9x + 3 \\
\hline
2x^2+7x-1 & 10x^4 + 17x^3 - 62x^2 + 30x - 3 \\
 & \underline{\underset{-}{10x^4} \underset{-}{+} 35x^3 \underset{+}{-} 5x^2} \\
 & \quad -18x^3 - 57x^2 + 30x - 3 \\
 & \quad \underline{\underset{+}{-18x^3} \underset{+}{-} 63x^2 \underset{-}{+} 9x} \\
 & \qquad 6x^2 + 21x - 3 \\
 & \qquad \underline{\underset{-}{6x^2} \underset{-}{+} 21x \underset{+}{-} 3} \\
 & \qquad\qquad \times
\end{array}
$$

Ans. : $5x^2 - 9x + 3$

(d)

$$
\begin{array}{r|l}
 & 4y^2 - 3 \\
\hline
3y-5 & 12y^3 - 20y^2 - 9y + 15 \\
 & \underline{\underset{-}{12y^3} \underset{+}{-} 20y^2} \\
 & \qquad - 9y + 15 \\
 & \qquad \underline{\underset{+}{-9y} \underset{-}{+} 15} \\
 & \qquad\qquad \times
\end{array}
$$

Ans. : $4y^2 - 3$

(e)

$$
\begin{array}{r|l}
 & 2y^2 + y - 1 \\
\hline
5y^2+3 & 10y^4+5y^3+y^2+4y - 2 \\
 & \underline{-10y^4 \quad -+6y^2} \\
 & +5y^3 - 5y^2 + 4y - 2 \\
 & \underline{\pm 5y^3 + 0 \quad \pm 3y} \\
 & -5y^2 + y - 2 \\
 & \underline{\mp 5y \quad + -3} \\
 & y + 1
\end{array}
$$

Ans.: $Q = 2y^2+y-1$
$R = y + 1$

(f)

$$
\begin{array}{r|l}
 & x^2+2x+1 \\
\hline
x^2+2-3x & x^4-x^3-3x^2+x+2 \\
 & \underline{-x^4 \; + -3x^3 \; - +2x^2} \\
 & 2x^3 - 5x^2+x \\
 & \underline{\underline{2x^3} \; - + \; 6x^2 \; \underline{-} +4x} \\
 & x^2 - 3x + 2 \\
 & \underline{\underline{x^2} \; - + \; 3x \; \underline{+} \; 2} \\
 & \times
\end{array}
$$

Ans. : x^2+2x+1

(g)

$$
\begin{array}{r|l}
 & 3x^3 - 5x + \frac{3}{2} \\
\hline
2x^2-6 & 6x^5-28x^3+3x^2+30x-9 \\
 & \underline{\underline{6x^5} \; + -18x^3} \\
 & -10x^3+3x^2+30x \\
 & \underline{\underline{-10x^3} \; + \; \pm 30x} \\
 & 3x^2-9 \\
 & \underline{\underline{3x^2} \; - + \; 9} \\
 & \times
\end{array}
$$

Ans. : $3x^3-5x + \frac{3}{2}$

(h)

$$
\begin{array}{r|l}
 & a - 2 \\
\hline
a^2-12a+13 & a^3-14a^2+37a-26 \\
 & \underline{\underline{a^3} \; - + \; -12a^2 \; \underline{-} +13a} \\
 & -2a^2+24a-26 \\
 & \underline{\underline{-2a^2} \; + - \; +24a \; \underline{+} -26} \\
 & \times
\end{array}
$$

Ans. : $a - 2$

(i)

$$
\begin{array}{r|l}
 & 3x^3+2x^2-9x-\frac{1}{2} \\
\hline
2x^2-3 & 6x^5+4x^4-27x^3-7x^2+27x+\frac{3}{2} \\
 & \underline{6x^5 \quad \mp 9x^3} \\
 & 4x^4-18x^3-7x^2 \\
 & \underline{4x^4 \quad \mp 6x^2} \\
 & -18x^3-x^2+27x \\
 & \underline{-18x^3 \quad \pm 27x} \\
 & -x^2+3/2 \\
 & \underline{-x^2 \pm 3/2} \\
 & \times
\end{array}
$$

Ans. : $3x^3+2x^2-9x-\frac{1}{2}$

(j)

$$
\begin{array}{r|l}
 & y^3-4y^2+19y-65 \\
\hline
y^2+4y+2 & y^5+5y^3+3y^2+5y+3 \\
 & \underline{y^5+2y^3+4y^4} \\
 & -4y^4+3y^3+3y^2 \\
 & \underline{\mp 4y^4 \mp 16y^3 \mp 8y^2} \\
 & 19y^3+11y^2+5y \\
 & \underline{19y^3+76y^2+38y} \\
 & -65y^2-33y+3 \\
 & \underline{\mp 65y^2 \mp 260y \mp 130} \\
 & 227y+133
\end{array}
$$

Ans. :
Q = $y^3-4y^2+19y-65$
R= $227y + 133$

4. (a)

$$
\begin{array}{r|l}
 & x^3-x^2+3x-5 \\
\hline
x+5 & x^4+4x^3-2x^2+10x-25 \\
 & \underline{x^4+5x^3} \\
 & -x^3-2x^2 \\
 & \underline{\mp x^3 \mp 5x^2} \\
 & 3x^2+10x \\
 & \underline{3x^2+15x} \\
 & -5x-25 \\
 & \underline{-5x-25} \\
 & \times
\end{array}
$$

Ans. : x^3-x^2+3x-5; Yes

(b)

$$
\begin{array}{r|l}
 & t^3-1 \\
\hline
t^3+1 & t^6+3t^2+10 \\
 & \underline{t^6 + t^3} \\
 & -t^3+3t^3+10 \\
 & \underline{-t^3 \quad -1} \\
 & \underline{+ \qquad +} \\
 & 3t^2+11
\end{array}
$$

Ans. : t^3-1, $3t^2+11$; No

(c)

$$
\begin{array}{r|l}
 & 3y^3-5y+3/2 \\
\hline
2y^2-6 & 6y^5-28y^3+3y^2+30y-9 \\
 & \underline{6y^5-18y^3} \\
 & \underline{- \quad +} \\
 & -10y^3+3y^2+30y \\
 & \underline{-10y^3+30y} \\
 & \underline{+ \quad -} \\
 & 3y^2-9 \\
 & \underline{3y^2-9} \\
 & \underline{- \quad \times}
\end{array}
$$

Ans. : $3y^2-5y+\frac{3}{2}$, Yes

5.

$$
\begin{array}{r|l}
 & x^3+x^2-1 \\
\hline
2x-3 & 2x^4-x^3-3x^2-2x+a \\
 & \underline{2x^4-3x^3} \\
 & \underline{- \quad +} \\
 & 2x^3-3x^2 \\
 & \underline{2x^3-3x^2} \\
 & \underline{- \quad +} \\
 & -2x+a \\
 & \underline{-2x+3} \\
 & \underline{+ \quad -} \\
 & a-3
\end{array}
$$

as, $a-3=0$; $a=3$

6.

$$
\begin{array}{r|l}
 & x^3+x^2-x+5 \\
\hline
3x+2 & 3x^4+5x^3-x^2+13x+9 \\
 & \underline{3x^4+2x^3} \\
 & \quad 3x^3-x^2 \\
 & \quad \underline{3x^3+2x^2} \\
 & \qquad -3x^2+13x+9 \\
 & \qquad \underline{-3x^2-2x} \\
 & \qquad\quad 15x+9 \\
 & \qquad\quad \underline{15x+10} \\
 & \qquad\qquad -1
\end{array}
$$

Dividend $= \text{Divisor (Quotient)} + \text{Remainder}$

$= 3x+2\,(x^3+x^2+x+5)-1$

$= 3x^4+3x^3-3x^2+15x+2x^2+2x^2-2x+10-1$

$= 3x^4+5x^3-x^2+13x+9$

(Hence verified)

7.

$$
\begin{array}{r|l}
 & 2x^2+2x-1 \\
\hline
4x^2+3x-2 & 8x^4+14x^3-2x^2+7x-8 \\
 & \underline{8x^4+6x^3-4x^2} \\
 & \quad 8x^3+2x^2+7x \\
 & \quad \underline{8x^3+6x^2-4x} \\
 & \qquad -4x^2+11x-8 \\
 & \qquad \underline{-4x^2-3x+2} \\
 & \qquad\qquad 14x-10
\end{array}
$$

So, $14x-10$ should be subtracted.

Exercise 5C

1. (a) $(3x+5)^2$

$(3x)^2+(5)^2+2\times 3x\times 5$

$9x^2+25+30x$

(b) $(y-9)^2$

$(y)^2+(9)^2-2y\times 9$

$y^2+81-18y$

(c) $(5x^2-4y^2)^2$

$(5x^2)^2+(4y^2)^2-2(5x^2)(4y^2)$

$25x^4+16y^4-40x^2y^2$

(d) $(7x-\frac{1}{2}y)^2$

$(7x)^2+(1/2y)^2-2\times\frac{1}{2}y\times 7x$

$49x^2+\frac{1}{4}y^2-7xy$

(e) $(2x+\frac{3}{2}x)^2$

$(2x)^2+\left(\frac{3}{x}\right)^2+2\times 2x\times\frac{3}{x}$

$4x^2+\frac{9}{x^2}+12$

(f) $(5ab-6cd)^2$

$(5ab)^2+(6cd)^2-2(5ab)(6cd)$

$25a^2b^2+36c^2d^2-60abcd$

(g) $\left(\frac{3x}{4}-\frac{4y}{5}\right)^2\Rightarrow\left(\frac{3x}{4}\right)^2+\left(\frac{4y}{5}\right)^2-2\left(\frac{3x}{4}\right)\left(\frac{4y}{5}\right)$

$\Rightarrow\frac{9x^2}{16}+\frac{16y^2}{25}-\frac{6xy}{5}$

2. (a) $81a^2+9b^2-54ab$

$\Rightarrow (9a)^2+(3b)^2-2\times 9a\times 3b$

$\Rightarrow (9a-3b)^2$; putting values

$\Rightarrow [9\times(-1)-3(-4)]^2$

$\Rightarrow [-9+12]^2$

$\Rightarrow (+3)^2=9$

(b) $36x^2+49y^2+84xy$

$(6x)^2+(7y)^2+2\times 6x\times 7y$

$$\begin{aligned}(6x+7y)^2 &= [6\times 3+7\times 6]^2\\ &= [18+42]^2\\ &= [60]^2\\ &= 360\end{aligned}$$

(c) $25x^2+16y^2-40xy$

$(5x)^2+(4y)^2-2\times 5x\times 4y$

$(5x-4y)^2$

$(5\times 6-4\times 7)^2$

$(30-28)^2$

$=4$

(d) $4x^2+\dfrac{9}{x^2}-12$

$(4)x^2+\left(\dfrac{3}{x}\right)^2-2\times 2x\times\dfrac{3}{x}$

$\left(2x-\dfrac{3}{x}\right)^2$

$\left(2\times 2-\dfrac{3}{2}\right)^2$

$\left(4-\dfrac{3}{2}\right)^2$

$\left(\dfrac{5}{2}\right)^2$

$=\dfrac{25}{4}$

3. $x - \frac{1}{x} = 5$ Squaring both sides

$$x^2 + \left(\frac{1}{x}\right)^2 - 2 \times x \times \frac{1}{x} = 25$$

$$x^2 + \frac{1}{x^2} = 27$$

Squaring both side

4. $$x^4 + \frac{1}{x^4} + 2 = 729$$

(Adding & subtracting 2)

$$x^4 + \frac{1}{x^4} = 727$$

$$x^2 + \frac{1}{x^2} = 23$$

$$x^2 + \frac{1}{x^2} + 2 - 2 = 23$$

5. $$\left(x + \frac{1}{x}\right)^2 = 25$$ equation (i)

$$x + \frac{1}{x} = 5$$

(Add & Subtract 2) =

$$x^2 + \frac{1}{x^2} = 38 \quad = 36 \quad x^2 + \frac{1}{x^2} + 2 - 2 = 38$$

$$\left(x - \frac{1}{x}\right)^2 = 6$$

squaring eq. (i) $x - \frac{1}{x}$

$$= 1444$$

$$x^4 + \frac{1}{x^4} + 2 = 1442$$

$$x^4 + \frac{1}{x^4}$$

6. (a) $(103)^2 \Rightarrow (100+3)^2 \Rightarrow (100)^2 + (3)^2 + 2(100)(3) = 10609$

(b) $(98)^2 \Rightarrow (100-2)^2 \Rightarrow (100)^2 + (2)^2 - 2(100)(2) = 9604$

(c) $(10.3)^2 \Rightarrow (10+0.3)^2 \Rightarrow (10)^2 + (0.3)^2 - 2(10)(0.3) = 106.09$

(d) $(99.5)^2 \Rightarrow (100-0.5)^2 \Rightarrow (100)^2 + (0.5)^2 - 2(100)(0.5) = 9900.25$

7. (a) $176 \times 176 - 124 \times 124 \Rightarrow (176)^2 - (124)^2 \Rightarrow (176+124)(176-124)$

$\Rightarrow 15600$

(b) $0.68 \times 0.68 - 0.32 \times 0.32 \Rightarrow (0.68)^2 - (0.32)^2$

$(0.68+0.32)(0.68-0.32) \Rightarrow 0.36$

(c) $1.06 \times 1.06 - 2 \times 1.06 \times 0.06 + 0.06 \times 0.06 \Rightarrow (1.06-0.06)^2 \Rightarrow 1$

(d) $\dfrac{23.71 \times 23.74 - 16.29 \times 16.29}{0.742} = \dfrac{(23.71)^2 - (16.29)^2}{0.742}$

$\dfrac{(23.71+16.29)(23.71-16.29)}{0.742} = 400$

8. (a) $(3x + 2y + 4z)^2 \Rightarrow (3x)^2 + (2y)^2 + (4z)^2 + 2(3x)(2y) + 2(2y)(4z) + 2(4z)(3x) \Rightarrow 9x^2 + 4y^2 + 16z^2 + 12xy + 16yz + 24xz$

(b) $(2x - y + 3z)^2 \Rightarrow (2x)^2 + (y)^2 + (3z)^2 - 2(2x)(y) - 2(y)(3z) + 2(3z)(2x) \Rightarrow 4x^2 + y^2 + 9z^2 - 4xy - 6yz + 12xz$

(c) $(x - 2y - 5z)^2 \Rightarrow (x)^2 + (2y)^2 + (5z)^2 - 2(x)(2y) + 2(2y)(5z) - 2(5z)(x) \Rightarrow x^2 + 4y^2 + 25z^2 - 4xy + 20yz - 10xz$

(d) $(5 + 4a - 8b)^2 \Rightarrow (5)^2 + (4a)^2 + (8b)^2 + 2(5)(4a) - 2(4a)(8b) - 2(8b)(5) \Rightarrow 25 + 16a^2 + 64b^2 + 40a - 64ab - 80b$

(e) $(x^2 + y^2 + z^2) \Rightarrow (x^2)^2 + (y^2)^2 + (z^2)^2 + 2(x^2)(y^2) + 2(y^2)(z^2) + 2(z^2)(x^2) \Rightarrow x^4 + y^4 + z^4 + 2x^2y^2 + 2y^2z^2 + 2z^2x^2$

(f) $(xy + yz + zx)^2 \Rightarrow (xy)^2 + (yz)^2 + (zx)^2 + 2(xy)(yz) + 2(yz)(zx) + 2(zx)(xy) \Rightarrow x^2y^2 + y^2z^2 + z^2x^2 + 2xy^2z^2 + 2yz^2x + 2zx^2y$

9. $(4)^2 + 4(5)^2 + 9(3)^2 + 4(4 \times 5) + 12(5 \times 3) + 2(3 \times 4)$ or $(x + 2y + 3z)^2$ or $(4 + 10 + 3 \times 3)^2 = 529$

10. $4x^2 + (y)^2 + 25z^2 + 4xy - 10yz - 20xz$

$$(2x + y - 5z)^2$$
$$(2 \times 4 + 3 - 5 \times 2)^2$$
$$(8 + 3 - 10)^2 = (1)^2 = 1$$

11. $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$x^2 + y^2 + z^2 + 2(xy + yz + zx)$

as $(x + y + z) = 9$

$xy + yz + zx = 23$

Substituting values

$$(9)^2 = x^2 + y^2 + z^2 + 2(23)$$
$$x^2 + y^2 + z^2 = 81 - 46 = 35$$

12. $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$x^2 + y^2 + z^2 + 2(xy + yz + zx)$

as $(x + y + z) = 15$

$$(x^2 + y^2 + z^2) = 77$$
$$(15)^2 = 77 + 2(xy + yz + zx)$$
$$\frac{225 - 77}{2} = xy + yz + zx$$
$$xy + yz + zx = 74$$

Exercise 5D

1. (a) $(5x + 9y)^3 = (5x)^3 + (9y)^3 + 3(5x)(9y)(5x + 9y)$

$125x^3 + 729y^2 + 675x^2y + 1215xy^2$

(b) $(3p - 2q)^3 = (3p)^3 - (2q)^3 - 3(3p)(2q)(3p - 2q)$

$27p^3 - 8q^3 - 54p^2q^2 + 36pq^2$

(c) $(x + 1)^3 = (x)^3 + (1)^3 + 3(x)(1)(x+1)$

$x^3 + 1 + 3x^2 + 3x$

(d) $(x - 1)^3 = (x)^3 - (1)^3 - 3(x)(1)(x - 1)$

$x^3 - 1 - 3x^2 + 3x$

(e) $\left(\frac{2}{3}a+\frac{5}{3}b\right)^2=\left(\frac{2}{3}a\right)^3+\left(\frac{5}{3}b\right)^3+3\left(\frac{2}{3}a\right)\left(\frac{5}{3}b\right)$

$\left(\frac{2}{3}a+\frac{5}{3}b\right)$

$\frac{8}{27}a^3+\frac{125}{27}b^3+\frac{20a^2b}{9}+\frac{50ab^2}{9}$

(f) $\left(2x+\frac{3}{x}\right)^3 \Rightarrow (2x)^3+\left(\frac{3}{x}\right)^3+3(2x)\left(\frac{3}{x}\right)\left(2x+\frac{3}{x}\right)$

$8x^3+\frac{27}{x^3}+36x+\frac{54}{x}$

(g) $\left(2x+\frac{1}{4y}\right)^3 \Rightarrow (2x)^3+\left(\frac{1}{4y}\right)^3+3(2x)\left(\frac{1}{4y}\right)\left(2x+\frac{1}{4y}\right)$

$8x^3+\frac{1}{64y^3}+\frac{3x^2}{y}+\frac{3x}{8y^2}$

2. (a) $a^3+8b^3,\ ab=15,\ (a+2b)=10$

$(a+2b)=10$, cubing both sides

$(a+2b)^3=10$

$a^3+8b^3+6ab\,(a+2b)=1000$

$a^3+8b^3+6\times 15(10)=1000$

$a^3+8b^3=1000-900=100$

(b) a^3-b^3 if $a-b=-8,\ ab=-12$

$a-b=-8$, or $(a-b)^3=(-8)^3$

$a^3-b^3-3ab(a-b)=-512$

$a^3-b^3-3\times -12(-8)=-512$

$a^3-b^3=-512+288$

$a^3-b^3=-224$

(c) x^3-y^3, if $x-y=\frac{10}{9}$ and $xy=\frac{5}{3}$

$x-y=\frac{10}{9}$

$$x^3 - y^3 - 3 \times \frac{5}{3}\left(\frac{10}{9}\right) = \frac{1000}{729}$$

$$x^3 - y^3 = \frac{1000}{729} + \frac{50}{9} = \frac{5050}{729}$$

(d) $64a^3 - 125b^3$; $4a - 5b = 16$; $ab = 12$; $4a - 5b = 16$

$(4a)^3 - (5b)^3 - 3 \times 4a \times 5b\,(4a - 5b) = 4096$

$64a^3 - 125b^3 - 60 \times 12 \times 16 = 4096$

$64a^3 - 125b^3 = 4096 + 11520 = 15616$

(e) $27x^3 - 8y^3$; if $3x - 2y = 5$; $xy = 1$

$(3x)^3 - (2y)^3 - 3(3x)(2y)(3x - 2y) = 125$

$27x^3 - 8y^3 - 18 \times 1(5) = 125$

$27x^3 - 8y^3 = 125 + 18 \times 5$

$27x^3 - 8y^3 = 215$

(f) $a^3 - \frac{1}{a^3}$; as $a - \frac{1}{a} = 5$

$$a^3 - \frac{1}{a^3} - 3a \times \frac{1}{a}\left(a - \frac{1}{a}\right) = (5)^3 = a^3 - \frac{1}{a^3} = 125 + 15 = 140$$

(g) $a^3 + \frac{1}{a^3}$; if $a + \frac{1}{a} = 4$

$$a^3 + \frac{1}{a^3} + 3a \times \frac{1}{a}\left(a + \frac{1}{a}\right) = (4)^3 = a^3 + \frac{1}{a^3} = 64 - 12 = 52$$

(h) $a^3 + \frac{1}{a^3}$; if $a + \frac{1}{a} = 6$

$$a^3 + \frac{1}{a^3} + 3a \times \frac{1}{a}\left(a + \frac{1}{a}\right) = 216$$

$$\Rightarrow a^3 + \frac{1}{a^3} = 216 - 18 = 198$$

(i) $8a^3 - 27b^3 - 18ab\,(2a - 3b)$ where $a = 8, b = 5$

or $(2a - 3b)^3 = ? \implies (2 \times 8 - 3 \times 5)^3$

$(16 - 15)^3 = (1)^3 = 1$

3. (a) $(1001)^3 = (1000 + 1)^3 = (1000)^3 + (1)^3 + 3(1000)\,(1000 + 1) = 1003003001$

(b) $(97)^3 = (100 - 3)^3 = (100)^3 - (3)^3 - 3(100)\,(3)\,(100 - 3) = 912673$

(c) $(598)^3 = (600 - 2)^3 = (600)^3 - (2)^3 - 3(600)\,(2)\,(600 - 2) = 213847192$

(d) $(9.9)^2 = (10 - 0.1)^2 = (10)^3 - (0.1)^3 - 3(10)(0.1)\,(10 - 0.1) = 970.299$

(e) $(10.2)^3 = (10 + .02)^3 = (10)^3 + (0.2)^3 + 3(10)(0.2)\,(10 + 0.2) = 1061.208$

4. (a) $(2x + 5)^3 - (2x - 5)^3$

$[(2x)^3 + (573 + 3 \times 2x \times 5\,(2x + 5)] - [(2x)^3 - (5)^3 - 3\,(2x)\,(2x - 5)]$

$120x^2 + 250$

(b) $(3x)^3 + (8y)^3 + 3 \times 3x \times 8y(3x+8y) - [(3x)^3 - (8y)^3 - 3 \times 3x \times 8y\,(3x - 8y)]$

$27x^3 + 512y^3 + 72xy(3x + 8y) - 27x^3 + 512y^3 + 72xy\,(3x - 8y)$

$1024y^3 + 216xy + 576xy^2 + 216xy - 576xy^2$

$1024y^3 + 432xy$

(c) $\left(\frac{x}{2} + \frac{y}{3}\right)^3 - \left(\frac{x}{2} - \frac{y}{3}\right)^3$

$$\left(\frac{x}{2}\right)^3 + \left(\frac{y}{3}\right)^3 + 3\left(\frac{x}{2}\right)\left(\frac{y}{3}\right)\left(\frac{x}{2} + \frac{y}{3}\right) - \left[\left(\frac{x}{2}\right)^3 - \left(\frac{y}{3}\right)^3 - 3\left(\frac{x}{2}\right)\left(\frac{y}{3}\right)\left(\frac{x}{2} - \frac{y}{3}\right)\right]$$

$$\left(\frac{x}{2}\right)^3+\left(\frac{y}{3}\right)^3+3\left(\frac{x}{2}\right)^2\left(\frac{y}{3}\right)+3\left(\frac{x}{2}\right)\left(\frac{y}{3}\right)^2-$$

$$\left(\frac{x}{2}\right)^3-\left(\frac{y}{3}\right)^3-3\left(\frac{x}{2}\right)^2\left(\frac{y}{3}\right)+3\left[\frac{x}{2}\right]\left[\frac{y}{3}\right]^2=\frac{2y^3}{27}+\frac{x^2y}{2}$$

(d) $\left[\frac{p}{2}-\frac{q}{3}\right]^3-\left(\frac{p}{2}+\frac{q}{3}\right)^3$

$$\left[\left(\frac{p}{2}\right)^3-\left(\frac{q}{3}\right)^3-3\left(\frac{p}{2}\right)\left(\frac{q}{3}\right)\left(\frac{p}{2}-\frac{q}{3}\right)\right]$$

$$-\left[\left(\frac{p}{2}\right)^3+\left(\frac{q}{3}\right)^3+3\left(\frac{p}{2}\right)\left(\frac{q}{3}\right)\left(\frac{p}{2}+\frac{q}{3}\right)\right]$$

$$\left(\frac{p}{2}\right)^3-\left(\frac{q}{3}\right)^3-3\left(\frac{p}{2}\right)^2\left(\frac{q}{3}\right)+3\left(\frac{p}{2}\right)\left(\frac{q}{3}\right)^2-$$

$$\left[\left(\frac{p}{2}\right)^3+\left(\frac{q}{3}\right)^3+3\left(\frac{p}{2}\right)^2\left(\frac{q}{3}\right)\right]=\frac{-p^2q}{2}$$

Exercise 5E

1. (a) $(x+5)(x+3)$
$x^2+(5+3)x+5\times 3$
$x^2+8x+15$

(b) $(x-3)(x-5)$
$x^2-(5+3)x+5\times 3$
$x^2-8x+15$

(c) $(x-5)(x-3)$
$x^2-(5+3)x+5\times 3$
$x^2-8x+15$

(d) $(x-2)(x+9)$
$x^2+(9-2)x+9\times -2$
$x^2+7x-18$

(e) $(x+2)(x-9)$
$x^2+(-9+2)x-9\times 2$
$x^2-7x-18$

(f) $(x-5)(x-7)$
$x^2-(5+7)x+5\times 7$
$x^2-12x+35$

(g) $(3x+5)(3x-7)$
$(3x)^2-7\times 3x+5\times 3x-7\times 5$
$9x^2-21x+15x-35$
$9x^2-6x-35$

(h) $(2x-3)(2x-5)$

$4x^2-6x-10x+15$

$4x^2-16x+15$

(i) $\left(a+\frac{4}{3}\right)+\left(a+\frac{1}{3}\right) \Rightarrow a^2+\left(\frac{4}{3}+\frac{1}{3}\right)a+\frac{4}{3}\times\frac{1}{3}$

$\Rightarrow a^2+\frac{5}{3}a+\frac{4}{9}$

2. (a) $(101\times 103)=(100+1)(100+3)$

$(100)^2+(1+3)100+(1\times 3)=10403$

(b) $97\times 102=(100-3)(100+2)$

$(100)^2+(2-3)100+(2\times -3)=9894$

(c) $96\times 105=(100-4)(100+5)$

$(100)^2+(-4+5)50(-4\times 5)=10080$

(d) $51\times 53=(50+1)(50+3)$

$(50)^2+(3+1)50+(3\times 1)=2703$

(e) $51\times 48=(50+1)(50-2)$

$(50)^2+(1-2)50+(1\times -2)=2448$

(f) $103\times 96=(100+3)(100-4)$

$(100)^2+(-4+3)100+(-4\times 3)=9888$

3. (a) $(x-4)(x^2+4x+16)$

$x^3-(4)^3=x^3-64$

(b) $(5-x)(25+5x+x^2)$

$(5)^3-(x)^3=125-x^3$

(c) $(2x-5y)(4x^2+10xy+25y^2)$

$(2x)^3-(5y)^3=8x^3-125y^3$

(d) $(3z-1)(9z^2+3z+1)$

$(3z)^2-(1)^3=27z^3-1$

(e) $(5x+2y)(25x^2-10xy+4y^2)$

$(5x)^3+(2y)^3=125x^3+8y^3$

(f) $\left(2+\frac{5}{x}\right)\left(4-\frac{10}{x}+\frac{25}{x^2}\right)$

$$(2)^3+\left(\frac{5}{x}\right)^3=8+\frac{125}{x^3}$$

(g) $\left(3x-\frac{y}{2}\right)\left(9x^2+\frac{3}{2}xy+\frac{1}{4}y^2\right)$

$$(3x)^3-\left(\frac{y}{2}\right)^3=27x^3-\frac{y^3}{8}$$

4. (a) $(x+5)(x^2-5x+25)-(x+4)(x^2-4x+16)$

$(x)^3+(5)^3-[(x)^3-(4)^3]$

$+125-(+64)=125-64=61$

(b) $(x+5)(x-4)+(x-4)(x-3)-(x+3)(x-2)$

$x^2+(5-4)x-20+x^2-(4+3)x+1^2-x^2-(3-2)x+6$

x^2-7x-2

6. $a+b+c=9;\ ab+bc+ac=26$

squaring both sides

$a^2+b^2+c^2+2(ab+bc+ac)=81$

$a^2+b^2+c^2+2\times 26=81$

$a^2+b^2+c^2=81-52=29$

$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ac)$

$=9(29-26)$

$=9\times 3=27$

7. $a+b+c=15$

$a^2+b^2+c^2+2(ab+bc+ca)=225$

$83+2(ab+bc+ca)=225$

$2(ab+bc+ca)=225-83$

$2(ab+bc+ca)=142$

$ab+bc+ca=71$

$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$(a+b+c)(a^2+b^2+c^2)-(ab+bc+ca)$

$(15)(83-71)=180$

8. $a+b+c=0$ and $3abc=27$

$a^3+b^3+c^3=3abc$ $\quad\because a+b+c=0$

$a^3+b^3+c^3=27$

9. (a) $(28)^3-(78)^3+(50)^3=3\times(28)(-78)(50)$
$=-327600$

(b) $(55)^3-(75)^3+(20)^3=3\times(55)\times(-75)\times(20)$
$=-247500$

10. (a) $(a+b+c)(a^2+b^2+c^2-ab-bc-ac)$
$a^2+b^2-3abc+c^3$

(b) $(2x+3y+4z)(4x^2+9y^2+16z^2-6xy-12yz-8xz)$
$8x^3+27y^3+64z^3-72xyz$

Check Your Mental Maths IQ

1. Degree of polynomial
2. Highest
3. 2
4. $x-y$
5. $(x-y)(x+y)(x^2+y^2)$
$(x^2-y^2)(x^2+y^2)=x^4-y^4$
6. a^3-b^3
7. $P=2(\ell+b)$

$$\text{Perimeter} = 2(4p^2-2r+1+3p^2-2r+2)$$
$$= 2(7p^2-4r+3)=14p^2-8r+3$$

8.

$$\begin{array}{r|l} & 4x-3 \\ \hline 4x-3 & 16x^2-24x+a \\ & \underline{16x^2-12x} \\ & \quad\; \underset{-}{} \quad \underset{+}{} \\ \hline & \quad -12x+a \\ & \quad \underline{-12x+9} \\ & \quad\; \underset{+}{} \quad \underset{-}{} \\ \hline & \quad\; a-9 \\ \hline \end{array}$$

$\because a-9=0$
$a=9$

Multiple Choice Questions

1. (c)
2. (a)
3. (a)
4. (c)
5. (b)
6. (c)
7. (a)
8. (a)

6 Factorization of Algebraic Expressions

Exercise 6A

1. (a) $2x^2 + 5x$
   $x(2x+5)$

   (b) $3x^2 - 6xy^2$
   $3x(x-2y^2)$

   (c) $6x^2 + 8x^2y = 2x^2(3+4y)$

   (d) $12x^3y^4 + 16x^2y^5 - 4x^5y^2 = 4x^2y^2(3xy^2 + 4y^3 - x^3)$

   (e) $18a^3b^2 + 36ab^4 - 24a^2b^2 = 6ab^2(3a^2 + 6b^2 - 4ab)$

2. (a) $(7+3)(2x+5) = 10(2x+5)$

   (b) $(x+2)y + (x+2)x = (x+y)(x+2)$

   (c) $5a(2x+3y) - 2b(2x+3y) = (5a-2b)(2x+3y)$

   (d) $8(5x+9y)^2 + 12(5x+9y) = 4(5x+9y)[2(5x+9y)+3]$

   (e) $9(a-2b)^2 + 6(2b-a) = 3(a-2b)[3(a-2b)+2(-1)]$
   $= 3(a-2b)[3(a-2b)-2]$

   (f) $(x-2y)^2 - 4x + 8y = (x-2y)\{(x-2y)-4\}$

   (g) $2a + 6b - 3(a+3b)^2 = a + 3b\{2 - 3(a+3b)\}$

3. (a) $(x+y)(2x+3y) - (x+y)(x+1)$
   $x + y(2x+3y-x-1)$
   $x + y(x+3y-1)$

   (b) $(x+y)(2a+b) - (3x-2y)(2a+b)$
   $(2a+b)(x+y-3x+2y)$
   $(2a+b)(-2x+3y)$

   (c) $x^2 + xy + 8x + 8y$
   $x(x+y) + 8(x+y)$
   $(x+y)(x+8)$

   (d) $15ab + 15 + 9b + 25a$
   $15ab + 25a + 9b + 15$
   $5a(3b+5) + 3(3b+5)$
   $(5a+3)(3b+5)$

(e) $ax^2+by^2+bx^2+ay^2$

$ax^2+bx^2+by^2+ay^2$

$x^2(a+b)+y^2(a+b)$

$(x^2+y^2)(a+b)$

(f) $3ax-6ay-8by+4bx$

$3ax+4bx-6ay-8by$

$x(3a+4b)-2y(3a+4b)$

$(x-2y)(3a+4b)$

4. (a) $9a^2-16b^2=(3a)^2-(4b)^2=(3a+4b)(3a-4b)$

(b) $36a^2-(x-y)^2=(6a)^2-(x-y)^2=(6a+x-y)(6a-x+y)$

(c) $80a^2-45b^2=5[16a^2-9b^2]=[(4a)^2-(3b)^2]=5(4a+3b)(4a-3b)$

(d) $(3a-b)^2-9c^2=(3a-b)^2-(3c)^2=(3a-b+3c)(3a-b-3c)$

(e) $16x^2-81=(4x)^2-(9)^2=(4x-9)(4x+9)$

(f) $3a^4-48b^4=3[(a^2)^2-(4b^2)^2]=3[(a^2+4b^2)(a^2-4b^2)]=3(a+2b)(a-2b)(a^2+4b^2)$

(g) $16a^2b-\dfrac{b}{16a^2}=b\left(16a^2-\dfrac{1}{16a^2}\right)$

$b\left(4a+\dfrac{1}{4a}\right)\left(4a-\dfrac{1}{4a}\right)$

(h) $100(x+y)^2-81(a+b)^2$

$\{10(x+y)+9(a+b)\}\{10(x+y)-9(a+b)\}$

5. (a) $x^2+8x+16$

$x^2+4x+4x+16$

$x(x+4)+4(x+4)$

$(x+4)(x+4)$

(b) $4a^2-4a+1$

$4a^2-2a-2a+1$

$2a(2a-1)-1(2a-1)$

$(2a-1)(2a-1)$

(c) $4x^2+12xy+9y^2$

$4x^2+6xy+6xy+9y^2$

$2x(2x+3y)+3y(2x+3y)$

$(2x+3y)(2x+3y)$

(d) $x^4-10x^2y^2+25y^4$

$x^4-5x^2y^2-5x^2y^2+25y^4$

$x^2(x^2-5y^2)-5y^2(x^2-5y^2)$

$(x^2-5y^2)(x^2-5y^2)$

(e) $a^4-2a^2b^2+b^4$

(f) $\dfrac{x^2}{64}+\dfrac{y^2}{9}+\dfrac{xy}{12}$

$a^4 - a^2b^2 - a^2b^2 + b^4$

$a^2(a^2 - b^2) - b^2(a^2 - b^2)$

$(a^2 - b^2)(a^2 - b^2)$

$(a+b)(a-b)(a+b)(a-b)$

$\frac{x^2}{64} + \frac{xy}{24} + \frac{xy}{24} + \frac{y^2}{9}$

$\frac{x}{8}\left(\frac{x}{8} + \frac{y}{3}\right) + \frac{y}{3} + \left(\frac{x}{8} + \frac{y}{3}\right)$

$\left(\frac{x}{8} + \frac{y}{3}\right)\left(\frac{x}{8} + \frac{y}{3}\right)$

6. (a) $4x^2 + 9y^2 + z^2 + 12xy + 6yz + 4zx$

$(2x)^2 + (3y)^2 + (z)^2 + 2 \times 2x \times 3y + 2 \times 3y \times z + 2 \times z \times 2x$

$(2x + 3y + z)^2$

(b) $x^2 + 4y^2 + z^2 - 4xy + 4yz - 2xz$

$(-x)^2 + (2y)^2 + (z)^2 + 2 \times -x \times 2y + 2 \times 2y \times z + 2 \times -x \times z$

$(-x + 2y + z)^2$

(c) $a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$

$(-a)^2 + (b)^2 + (c)^2 + 2 \times -a \times b + 2 \times b \times c + 2 \times c \times -a$

$(-a + b + c)^2$

(d) $81x^2 + 4y^2 + z^2 + 36xy + 4yz + 18zx$

$(9x)^2 + (2y)^2 + (z)^2 + 2 \times 9x \times 2y + 2 \times 2y \times z + 2 \times 9x \times z$

$(9x + 2y + z)^2$

(e) $8x^3 + 36x^2y + 54xy^2 + 27y^3$

$(2x)^3 + (3y)^3 + 3 \times 2x \times 3y\,(2x + 3y)$

$(2x + 3y)^3$

(f) $8x^3 - 125 - 60x^2 + 150x$

$(2x)^3 - (5)^3 - 3 \times 2x \times 5\,(2x - 5)$

$(2x - 5)^3$

(g) $27a^3 - 64 - 108a^3 + 144a$

$(3a)^3 - (4)^3 - 3 \times 3a \times 4\,(3a - 4)$

$(3a - 4)^2$

Exercise 6B

1. (a) $x^2 + 6x + 8$

$x^2 + 4x + 2x + 8$

(b) $x^2 + 4x - 21$

$x^2 + 7x - 3x - 27$

$x(x+4)+2(x+4)$
$(x+2)(x+4)$

(c) $x^2-7x+12$
$x^2-4x-3x+12$
$x(x-4)-3(x-4)$
$(x-3)(x-4)$

(d) $x^2-23x+132$
$x^2-12x-11x+132$
$x(x-12)-11(x-12)$
$(x-11)(x-12)$

(e) $x^2-21x+108$
$x^2-12x-9x+108$
$x(x-12)-9(x-12)$
$x-9)(x-12)$

(f) $x^2+5x-36$
$x^2+9x-4x-36$
$x(x+9)-4(x+9)$
$(x-4)(x+9)$

(g) $-x^2+3x+40$
$-x^2+8x-5x+40$
$-x(x-8)-5(x-8)$
$(-x-5)(x-8)$

(h) $x^2-11x-42$
$x^2-14x+3x-42$
$x(x-14)+3(x-14)$
$(x+3)(x-14)$

(i) $a^2+19a-150$
$a^2+25a-6a-150$
$a(a+25)-6(a+25)$
$(a-6)(a+25)$

2. (a) $2x^2+5x+3$
as $2\times3=6, [2,3]$
$2x^2+2x+3x+3$
$2x(x+1)+3(x+1)$
$(2x+3)(x+1)$

(b) $6x^2+5x-6$
as $6\times-6=-36, [-4,9]$
$6x^2+9x-4x-6$
$3x(2x+3)-2(2x+3)$
$(3x-2)(2x+3)$

(c) $6x^2-13x+6$
as $6\times6=36, [-9,-4]$
$6x^2-9x-4x+6$
$3x(2x-3)-2(2x-3)$
$(3x-2)(2x-3)$

(d) $-2x^2-3+2$
as $2\times-2=-4,\ [-4,1]$
$-2x^2-4x+x+2$
$-2x(x+2)+1(x+2)$
$(1-2x)(x+2)$

(e) $12x^2-23xy+10y2$
$12\times10=120,\ [8,15]$
$12x^2-15xy-8x+10y^2$
$3x(4x-5y)-2(4x-5y)$

(f) $6x^2+35xy-6y^2$
as $6\times-6=-36, [36,-1]$
$6x^2+36xy-xy-6y^2$
$6x(x+6y)-y(x+6y)$

$(3x-2)(4x-5y)$ $\quad$ $(6x-y)(x+6y)$

(g) $3x^2-4x-4$

$4\times 3=12, [6, 2]$

$3x^2-6x+2x-4$

$3x(x-2)+2(x-2)$

$(3x+2)(x-2)$

(h) $11x^2-54x+63$

$63\times 11=693, [21, 33]$

$11x^2-33x-21x+63$

$11x(x-3)-21(x-3)$

$(11x-21)(x-3)$

(i) $9x^2-22xy+8y^2$ (product $=9\times 8=72$, $[18, 4]$

$9x^2-18xy-4xy+8y^2$

$9x(x-2y)-2y(x-2y)$

$(9x-2y)(x-2y)$

Exercise 6C

1. (a) $8x^3+125$

$(2x)^3+(5)^3$

$(2x+5)(4x^2-10x+25)$

(b) y^3+512

$(y)^3+(8)^3$

$(y+8)(y^2-8y+64)$

(c) $125a^3+34^3b^3$

$(5a)^3+(7b)^3$

$(5a+7b)(25a^2-35ab+49b^2)$

(d) x^6+y^6

$(x^2)^3+(y^2)^3$

$(x^2+y^2)(x^4-x^2y^2+y^4)$

(e) $128x^3+54y^3$

$2(64x^3+27y^3)$

$2(4x)^3+(3y^3)$

$2(4x+3y)(16x^2-12xy+9y^2)$

(f) $54a^6b+2a^3b^4$

$2a^3b(27a^3+b^3)$

$2a^3b\{(3a)^3+(b)^3\}$

$2a^3b(3a+b)(9a^2-3ab+b^2)$

(g) $1+27x^3$

$(1)^3+(3x)^3$

$(1+3x)(1+9x^2-9x)$

(h) $64x^3+y^3$

$(4x)^3+(y)^3$

$(4x+y)(16x^2+y^2-4xy)$

2. (a) $8x^3 - 343$

$(2x)^3 - (7)^3$

$(2x-7)(4x^2+49+14x)$

(b) $64x^3 - y^3$

$(4x)^3 - (y)^3$

$(4x+y)(16x^2+y^2-4xy)$

(c) $1 - 27x^3$

$(1)^3 - (3x)^3$

$(1-3x)(1+9x^2+9x)$

(d) $2x^4 - 128x$

$2x\{(x)^3 - (64)\}$

$2x\{x^3 - (4)^3\}$

$2x(x-4)(x^2+16+4x)$

(e) $x - 8xy^3$

$x(1-8y^3)$

$x\{(1)^3 - (2y)^3$

$x(1-2y)(1+4y^2+2y)$

(g) $\frac{x^3}{64} - 8y^3$

$\left(\frac{x}{4}\right)^3 - (2y)^3$

$\left(\frac{x}{4} - 2y\right)\left(\frac{x}{16} + 4y^2 + \frac{x}{2}y\right)$

(h) $a^4b - ab^4$

$ab(a^3 - b^3)$

$ab(a-b)(a^2+b^2+ab)$

3. (a) $(3x-2y)^3 + (2y-5z)^2 + (5z-3x)^3$

$3x - 2y = a, 2y - 5z = b, 5z - 3x = c$

Clearly $a + b + c = 0, a^3 + b^3 + c^3 = 3abc$

$3(3x-2y)(2y-5z)(5z-3x) = (3x-2y)^3 + (2y-5z)^3 + (5z-3x)^3$

(b) $p^3(q-r)^3 + q^3(r-p)^3 + r^3(p-q)^3$

$(pq-pr)^3 + (qr-qp)^3 + (rp-rq)^3$

Clearly $a + b + c = 0,\ a^3 + b^3 + c^3 = 3abc$

So, $3(pq-pr)(qr-qp)(rp-rq)$

(c) $a^3 + 8b^2 - 64c^3 + 24abc$

$(a)^3 + (2b)^3 + (-4c)^3 - 3 \times a \times 2b \times -4c$

$(a+2b-4c)(a^2+4b^2+16c^2-2ab+8bc-4ac)$

(d) $x^3 - 8y^3 - 64z^3 - 24xyz$

$(x)^3 + (-2y)^3 + (-4z)^3 - 3(-2y)(x)(-4z)$

$(x - 2y - 4z)(x^2 + 4y^2 + 16z^2 + 2xy - 8xz + 4zx)$

(e) $125 + 8x^3 - 27y^3 + 90xy$

$(5)^3 + (2x)^3 - (3y)^3 - 3 \times 5 \times 2x \times -3y$

$(5 + 2x - 3y)(25 + 4x^2 + 9y^2 - 10x + 6xy + 15y)$

4. $\dfrac{73 \times 73 \times 73 + 27 \times 27 \times 27}{73 \times 73 - 73 \times 27 + 27 \times 27} \Rightarrow \dfrac{(73)^3 + (27)^3}{(73)^2 - 73 \times 27 + (27)^2}$

$\dfrac{a^3 + b^3}{a^2 - ab + b^2}$, where $a = 73$; $b = 27$

$\Rightarrow \dfrac{(a+b)(a^2 - ab + b^2)}{(a^2 - ab + b^2)} = (a+b)$

$= (73 + 27) = 100$

(b) $\dfrac{135 \times 135 \times 135 - 65 \times 65 \times 65}{135 \times 135 + 135 \times 65 + 65 \times 65} \Rightarrow \dfrac{(135)^3 - (65)^3}{(135)^2 + 135 \times 65 + (65)^2}$

$\dfrac{a^3 - b^3}{a^2 - ab + b^2}$, where $a = 135$; $b = 65$

$\Rightarrow \dfrac{(a-b)(a^2 + ab + b^2)}{(a^2 + ab + b^2)} = (a+b)$

$= (135 - 65) = 70$

(c) $\dfrac{0.87 \times 0.87 \times 0.87 + 0.13 \times 0.13 \times 0.13}{0.87 \times 0.87 - 0.87 \times 0.13 + 0.13 \times 0.13}$

$\Rightarrow \dfrac{(0.87)^3 + (0.13)^3}{(0.87)^2 - 0.87 \times 0.13 + (0.13)^2}$

$(0.87 + 0.13) = 1$

(d) $\dfrac{27x^3 - 1000}{3x - 10} = \dfrac{(3x)^3 - (10)^3}{3x - 10} = \dfrac{(3x - 10)(9x^2 + 30x + 100)}{(3x - 10)}$

$= 9x^2 + 30x + 100$

(e) $\dfrac{343x^3-64}{49x^2+36x+16}=\dfrac{(7x)^3-(4)^3}{(7x)^2+7x\times 4+(4)^2}$

$$=\frac{(7x-4)(49x^2+28x+16)}{(7x+4)(7x+4)}$$

Mental Math I.Q.

A. 1. $3(a-4b)-9x(a-4b)=(3-9x)(a-4b)$

$=3(1-3x)(a-4b)$

2. $3a(x+y)+5b(x+y)-7c(x+y)=(x+y)(3a+5b-7c)$

3. $9x^2-6xy+y^2=(3x-y)^2=(3x-y)(3x-y)$

4. $16x^2-9y^2=(4x)^2-(3y)^2=(4x+3y)(4x-3y)$

5. $3a^3-3a=3a(a^2-1)=3a(a+1)(a-1)$

6. $\dfrac{1}{64}a^3-\dfrac{1}{216}b^3\Rightarrow\left(\dfrac{1}{4}a\right)^3-\left(\dfrac{1}{6}b\right)^3$

$$\Rightarrow\left(\frac{1}{4}a-\frac{1}{6}b\right)\left(\frac{a^2}{16}+\frac{b^2}{36}+\frac{ab}{24}\right)$$

7. $x^2+13x+40=x^2+8x+5x+40$

$x(x+8)+5(x+8)=(x+5)(x+8)$

8. a^4-81b^4

$(a^2)^2-(9b^2)^2$

$(a^2+9b^2)(a^2-9b^2)=(a^2+9b^2)[(a)^2-(3b)^2]$

$=(a^2+9b^2)(a+3b)(a-3b)$

Multiple Choice Questions

1. (a) $(x-1)(x+1)$ 2. (c) $(a+b)(m+n)$

3. (a) $4x$

4. (b) 27; as, $x-\dfrac{1}{x}=5\Rightarrow x^2+\dfrac{1}{x^2}=25+2=27$

5. (c) $(a-b)^2=(5)^2=a^2+b^2=25+2(3)=31$

6. (c) $(4x-5y)(4x+5y)$

7. (b) $81x^2-100$

$(9x)^2-(10)^2=(9x+10)(9x-10)$

8. (b) $\dfrac{(72)^2-(38)^2}{(72-38)}\Rightarrow\dfrac{(72+38)(72-38)}{(72-38)}=72+38=110$

7 Linear Equalions in One Variable

Exercise 7A

1. $8x = 20 + 3x$
   $8x - 3x = 20$
   $5x = 20$
   $x = 4$

2. $5x - 7 = 2x + 8$
   $5x - 2x = 8 + 7$
   $3x = 15$
   $x = 5$

3. $4x - 3 = 2x + 5$
   $4x - 2x = 5 + 3$
   $2x = 8$
   $x = 4$

4. $8x - 11 - 5x + 3 = 2x + 4 - 3x$
   $-11 + 3 - 4 = 2x - 3x + 5x - 8x$
   $-12 = -4x$
   $3 = x$

5. $5x - (3x - 1) = x - 4$
   $5x - 3x + 1 = x - 4$
   $5x - 3x - x = -4 - 1$
   $x = -5$

6. $10p - (3p - 4) = 4(p + 1)$
   $10p - 3p + 4 = 4p + 4$
   $10p - 3p - 4p = 4 - 4$
   $3p = 0$
   $p = 0$

7. $4(x + 3) - 2(x - 1) = 3x + 3$
   $4x + 12 - 2x + 2 = 3x + 3$
   $4x - 2x - 3x = 3 - 12 - 2$
   $-x = -11$
   $x = 11$

8. $\frac{2}{3}x + 1 = \frac{7}{3}$
   $\frac{2}{3}x = \frac{7}{3} - 1$
   $\frac{2}{3}x = \frac{7-3}{3} = \frac{4}{3}$
   $\frac{2}{3}x = \frac{4}{3}$
   $x = \frac{4 \times 3}{3 \times 2} = 2$

9. $\frac{1}{4}x + \frac{1}{6}x = x - 7$
   $\frac{3x + 2x - 12x}{12} = -7$
   $-7x = -7 \times 12 \Rightarrow x = 12$

10. $3x + \frac{2}{3}x = 2x + 1$
    $3x - 2x = 1 - \frac{2}{3}$
    $x = -\frac{1}{3}$

11. $\frac{3x}{4}+\frac{x}{6}=22$

$\frac{9x+2x}{12}=22$

$11x=22\times 12$

$x=\frac{22\times 12}{11}$

$x=2\times 12$

$x=24$

12. $\frac{x-3}{5}+\frac{x-4}{7}=6-\frac{2x-1}{35}$

$\frac{7x-21+5x-20+2x-1}{35}=6$

$14x-42=210$

$14x=252$

$x=18$

13. $\frac{4x+1}{3}+\frac{2x-1}{2}-\frac{3x-7}{5}=6$

$\frac{40x+10+30x-15-18x+42}{30}=6$

$52x+37=180$

$52x=143$

$x=\frac{143}{52}=\frac{11}{4}$ or $2\frac{3}{4}$

14. $\frac{0.5y-4}{2.4y+6}=\frac{-5}{3}$

$1.5y-12=-12y-30$

$13.5y=-18$

$y=\frac{-18}{13.5}=\frac{-4}{3}$

$y=-1\frac{1}{3}$

15. $\frac{5x-3}{2x}=\frac{8}{9}$

$45x-27=16x$

$29x=27$

$x=\frac{27}{29}$

16. $\frac{5+3x}{3-2x}=\frac{5}{3}$

$15+9x=15-10x$

$15-15=-19x$

$0=x$

17. $\frac{19-3y}{1-9y}=\frac{8}{5}$

$95-15y=8-72y$

$87=-57y$

$-\frac{29}{19}=y$

18. $\frac{3x+4}{2x+5}=\frac{1}{2}$

$6x+8=2x+5$

$4x=-3$

$x=-\frac{3}{4}$

19. $\dfrac{z/4-3/5}{4/3-7z}=\dfrac{-3}{20}$

$5z-12=-4+21z$

$-8=16z$

$\dfrac{-1}{2}=z$

20. $\dfrac{3x}{5x+2}=-4$

$3x=-20x-8$

$23x=-8$

$x=\dfrac{-8}{23}$

21. $0.26x+0.09x=8-0.45x$

$0.35x+0.45x=8$

$0.8x=8$

$x=10$

22. $\dfrac{0.5y+4}{1.2y+8}=\dfrac{5}{3}$

$1.5y+12=6y+40$

$-28=4.5y$

$\dfrac{-28}{4.5}=y$

or $y=\dfrac{-56}{9}$

23. $\dfrac{6x+7}{3x+2}=\dfrac{4x+5}{2x+3}$

$12x^2+14x+18x+21=12x^2+8x+15x+10$

$32x+21=23x+10$

$9x=-11$

$x=\dfrac{-11}{9}$

24. $\dfrac{2-7x}{1-5x}=\dfrac{3+7x}{4+5x}$

$8+10x-28x-35x^2$

$=3+7x-15x-35x^2$

$-18x+8=3-8x$

$10x=5$

$x=\dfrac{1}{2}$

25. $\dfrac{(4+x)(5-x)}{(2+x)(7-x)}=1$

$\dfrac{20-4x+5x-x^2}{14-2x+7x-x^2}=1$

$20+x-x^2=14+5x-x^2$

$6=4x$

$\dfrac{3}{2}=x$

26. (a) $\dfrac{x^2-9}{5+x^2}=\dfrac{-5}{9}$

(b) $\dfrac{2x^2-5}{x^2+2}=\dfrac{3}{2}$

$9x^2 - 81 = -25 - 5x^2$

$14x^2 = 56$

$x^2 = 4$

$x = 2$

$4x^2 - 10 = 3x^2 + 6$

$x^2 = 16$

$x = 4$

Exercise 7B

1. Let the number be x

   According to question

$$\frac{2}{3}x + 6 = 36$$

$$\frac{2}{3}x = 30$$

$$x = \frac{90}{2} = 45$$

$$x = 45$$

2. Let the numbers are n, n+2, n+4

   According to question

$$n + n + 2 + n + 4 = 246$$

$$3n = 246 - 6$$

$$n = \frac{240}{3}$$

$$n = 80$$

   So, the number are $n = 80$, $n + 2 = 82$, $n + 4 = 84$

3. Let the first part be x

   According to question

   2nd part $= 2x - 32$, 3rd part $= x + 18$

   So, $x + 2x - 32 + x + 18 = 534$

$$4x = 534 + 14$$

$$x = \frac{548}{4} = 137 = x$$

   Ist part 137, 2nd $= 2 \times 137 - 32 = 242$, 3rd $= 137 + 18 = 155$

4. Let the numbers be 5x and 8x

   So, $5x + 8x = 182$

$$13x = 182$$

$$x = \frac{182}{13}$$

$$x = 14$$

So numbers are :

$5 \times 14 = 70$; $\quad 8 \times 14 = 112$

5. Let the ten's digit be $= x$

Unit's digit $= (15 - x)$

Original number $= 10 \times$ ten's digit $+$ unit's digit

$= 10 \times x + 15 - x = 9x + 15$

On interchanging digits, new number

$= 10(15 - x) + x$

$= 150 - 9x$

According to question

$$9x + 15 - 150 + 9x = 27$$

$$18x = 27 - 15 + 150 = 162$$

$$x = \frac{162}{18} = 9$$

So, Ten's digit $= 9$; ones digit $= 6$

The number $= 9 \times 10 + 6 = 96$

6. Let the numerator be x

According to question; Denominator $x + 3$

As $\quad \dfrac{x-3}{x+2+3} = \dfrac{1}{5}$

$$5x - 15 = x + 5$$

$$4x = 20$$

$$x = 5$$

Numerator $= 5$, denominator $= 8$

Required fraction $= \dfrac{5}{8}$

7. Let the breadth be x

then length be $x + 9$

Area of given rectangle is $= x^2 + 9x$

Now, A.T.Q. Area of new rectangle is

$$(x+9+3)(x+3) = x^2 + 15x + 36$$

As $\quad x^2 + 9x + 84 = x^2 + 15x + 36$

$$84 - 36 = 15x - 9x$$

$$48 = 6x$$
$$8 = x$$

So, length $= 8 + 9 = 17$; breadth $= 8$

8. Let the ratio be x

as Rama age $= 3x$; Namita age $= 8x$

According to question

Rama age $= 3x - 3$. Namita age $= 8x + 2$

Then $(8x+2) = 6(3x-3)$

$$8x + 2 = 18x - 18$$
$$20 = 10x$$
$$2 = x$$

So, Rama age $= 3 \times 2 = 6$; Namita age $= 8 \times 2 = 16$

9. A purse has only 2-rupee and 5-rupee coins.

Let the 2 rupee coins be x and 5-rupee coins be y.

So, According to question

$$x + y = 48$$

and $2x + 5y = ₹132$

From eq. (i)

$$x = 48 - y$$
$$2(48 - y) + 5y = 132$$
$$96 - 2y + 5y = 132$$
$$3y = 36$$
$$y = 12$$

As, $x = 48 - 12$

$$x = 36$$

So, the 2 rupees coins are 36

10. The perimeter of a rectangle is 240 cm

Let the length and breadth be x, y

as $2(l + b) = 240$

$2(x + y) = 240$ or $x + y = 120$ .....(i)

$$2\left[\left(x - \frac{10x}{100}\right) + \left(y + \frac{20y}{100}\right)\right] = 240$$

$$x - \frac{10}{100}x + y + \frac{20}{100}y = 120$$

$$x + y - \frac{10x}{100} + \frac{20}{100}y = 120$$

$$9x + 12y = 1200, \quad \text{..... (i)}$$

$$x + y = 120 \quad \text{..... (ii)}$$

$$9x + 12y = 1200$$

$$\underline{-9x \pm 9y = 1080}$$

$$3y = 120$$

$$y = 40$$

$$x + y = 120$$

$$x + 40 = 120$$

$$x = 120 - 40 = 80$$

So, the length = 80 cm; the breadth = 40 cm

11. Given speed of the stream = 1.5 km/hr

Let the speed of motor boat = x km/hr

Speed of boat in the direction of stream = (x + 1.5) km/hr

Speed in opposite direction = (x–1.5) km/hr

Distance travelling in the direction of stream in 5 hour

$$= 5(x + 1.5)$$

Distance travelled in opposite direction in $\frac{11}{2}$ hr

$$= \frac{11}{2}(x - 1.5)$$

As per question

$$5(x + 1.5) = \frac{11}{2}(x - 1.5)$$

$$2(5x + 7.5) = 11x - 16.5$$

$$10x + 15 = 11x - 16.5$$

$$15 + 16.5 = 11x - 10x$$

$$x = 31.5$$

So, speed of the stream in still water = x = 31.5 km/h

12. Let total work be = x

Gorav complete x work in 18 hours

So, Gorav 1 hour work = $\frac{x}{18}$

Sorav 1 hour work = $\frac{x}{24}$

Sorav 2 hour work $= \frac{x}{24} \times 2 = \frac{x}{12}$

Remaining work $= \frac{x}{1} - \frac{x}{12} = \frac{11x}{12}$

Sorav and Gorav 1 hour work $= \frac{x}{18} + \frac{x}{24} = \frac{4x+3x}{72} = \frac{7x}{72}$

Both of them complete $\frac{7x}{72}$ work in 1 hour

Both of them complete 1 work in $= \frac{1 \times 72}{7x}$

Both of them complete $\frac{11x}{12}$ work $= \frac{1 \times 72}{7x} \times \frac{11x}{12} = \frac{66}{7}$ hrs.

or $9\frac{3}{7}$ hours

So, they will complete the work in $= 9\frac{3}{7} + 2 = 11\frac{3}{7}$ hours

Mental Math I.Q.

1. $$\frac{2y+1}{y+2} = \frac{4}{5}$$
$$10y + 5 = 4y + 8$$
$$6y = 3$$
$$y = \frac{1}{2}$$

2. $$0.3(6+m) = 0.4(8-m)$$
$$1.8 + 0.3m = 3.2 - 0.4m$$
$$0.7m = 1.4$$
$$m = 2$$

3. Let the ratio be x

$$3x + 4x + 5x = 180° \text{ (Angle sum property of triangle)}$$
$$12x = 180°$$
$$x = \frac{180}{12}$$
$$x = 15$$

So angles are $3 \times 15 = 45$

$4 \times 15 = 60$
$5 \times 15 = 75$

4. Let the ratio be x
   According to question $23x + 7x = 60$
   $30x = 60$
   $x = 2$
   So, numbers are $= 23 \times 2 = 46$; $7 \times 2 = 14$

Multiple Choice Questions

1. $2x + 7 = x + 9$
   $x = 2$ (c)

2. $6(y-3) - 3(y-7) = 0$
   $6y - 18 - 3y + 21 = 0$
   $3y - 3$
   $y = -1$ (b)

3. $\frac{7m+2}{5} = \frac{6m-5}{11}$
   $77m + 22 = 30m - 25$
   $47m = -47$
   $m = -1$ (b)

4. Let the number be x.
   $3x + 6 = 45$
   $3x = 39$
   $x = 13$ (a)

5. Let the number be x, x+2, x+4
   $x + x + 2 + x + 4 = 24$
   $3x = 18$
   $x = 6$
   Numbers are 6, 8, 10 (a)

6. Let the ratio be x
   $5x + 11x = 160$
   $x = 10$
   So numbers are, $5 \times 10 = 50$; $11 \times 10 = 110$

7. Let the ratio be x
   $5x - 2x = 60$
   $3x = 60$
   $x = 20$
   So numbers are, 40, 100 (a)

8. Let the son age be x; then Ajay age will be 3x; after 12 years
   $x + 12, 3x + 12$
   A.T.Q. $2(x + 12) = (3x + 12)$
   $2x + 24 = 3x + 12$
   $12 = x$
   So, Son age = 12; Ajay age = 36 (a)

Unit 3 : Ratio and Proportion

8 Percentage and Its Application

Exercise 8A

1. (a) 30%

$\frac{30}{100} = \frac{3}{10}$

or 3 : 10

(b) 2.5%

$\frac{25}{1000} = \frac{1}{40}$

1 : 40

(c) 0.75%

$\frac{75}{10000} = \frac{3}{4000}$

3 : 4000

(d) 125%

$\frac{125}{100} = \frac{5}{4}$

5 : 4

(e) $12\frac{1}{2}\% = \frac{25}{2}\%$

$\frac{25}{2 \times 100} = \frac{1}{8}$ or 1 : 8

2. (a) $\frac{3}{4} = \frac{3}{4} \times 100 = 75\%$

(b) $\frac{13}{75} = \frac{13}{75} \times 100 = 17\frac{1}{3}\%$

(c) $0.18 = \frac{18}{100} \times 100 = 18\%$

(d) $0.275 = \frac{275}{1000} \times 100 = 27.5\%$

(e) $1.25\% = \frac{1.25}{100} \times 100 = 125\%$

3. (a) $\frac{12}{100} \times 1200 =$ ₹144

(b) $\frac{13}{100} \times 6500 =$ ₹845

(c) $\frac{20}{100} \times 800 =$ ₹160

4. Decrease $= 60 - 45 = 15$

Decrease% $= \frac{15}{60} \times 100 = 25\%$

5. Let the cost be x

$x + \frac{20}{100} \times x = 600 \quad \Rightarrow \frac{x}{1} + \frac{x}{5} = 600$

$5x + x = 3000$

$x = \frac{3000}{5} = 600$

6. Candidates are A and B, Let the votes be = 100
   100 – 46 = 54% votes will be got by B
   As 54 – 46 = 8
   So the no of valid votes be

$$= \frac{100}{8} \times 1600$$

$$= 20{,}000 \text{ votes}$$

7. Let the total gunpowder is 100 g
   Out of it 75 nitre and 10 sulphure and remaining were

$$= 100 - 85 = 15\text{g}$$

   The amount of charcoal in 9000 g of given powder is

$$= \frac{15}{100} \times 9000$$

$$= 1350 \text{ g or } 1\text{kg } 35\text{gm}$$

8. Let the expenditure is ₹100
   So total be 30 + 3 = 33
   ∴ Total salary for the month would be

$$= \frac{2310}{33} \times 100 = ₹7000$$

9. Let the total money the man had = ₹100
   Spent on children = ₹40
   Remaining money = ₹ 100 – ₹ 40 = ₹ 60

   20% of remaining = $₹60 \times \frac{20}{100} = ₹12$

   So, now balance = ₹60 – ₹12 = ₹48

   So, actual money the man has = $\frac{100}{48} \times \frac{9600}{1} = ₹20000$

10. Let the cost of sugar be = ₹ 100
    Increased cost = ₹ 125

    So, consumption a housewife should decrease = $\frac{25}{125} \times 100 = 20\%$

11. Let the maximum marks be = 100x
    Fist student got = 25x marks

So pass marks $= 25x + 30$ .....(i)

Second student got $= 50x$ marks

So, pass marks $= 50x - 20$ ..... (ii)

Comparing (i) and (ii)

$$25x + 30 = 50x - 20$$
$$25x = 50; x = 2$$

So, the maximum marks $= 100 \times 2 = 200$ marks

So, minimum pass marks

$$= 25 \times 2 + 30$$
$$= 50 + 30 = 80 \text{ marks}$$

12. Let the original salary of the officer be $= 100$

Increased salary $= 100 + 50 = 150$

So, the salary % to be decreased $= \frac{50}{150} \times 100 = 33\frac{1}{3}\%$

Exercise 8B

1. C.P. of CD player = ₹ 1500

$$\text{S.P.} = 1750$$
$$\text{Gain} = \text{S.P.} - \text{C.P.}$$
$$= 1750 - 1500$$
$$= 250$$
$$\text{Gain \%} = \left(\frac{\text{Gain}}{\text{C.P.}} \times 100\right)\%$$
$$= \frac{250}{1500} \times 100 = 16\frac{2}{3}\%$$

2. C.P. = ₹2200; S.P. = ₹1980

$$\text{Loss} = ₹2200 - ₹1980$$
$$= ₹220$$
$$\text{Loss\%} = \left(\frac{\text{Loss}}{\text{C.P.}} \times 100\right)\%$$
$$= \frac{220}{2200} \times 100 = 10\%$$

3. Total C.P. = ₹12000 + ₹2850

= ₹14850

S.P. = ₹13860

As S.P. < C.P.

Loss = ₹14850 – ₹13860

= ₹ 90

$$\text{Loss \%} = \frac{990}{14850} \times 100 = 6\frac{2}{3}\%$$

4. Let the C.P. of 18 mangoes be x

The C.P. of 1 mango = $\frac{x}{18}$

But, S.P. of 16 mangoes = x

S.P. of 1 mango = $\frac{x}{16}$

$$\text{Profit} = \frac{x}{16} - \frac{x}{18} = \frac{x}{144}$$

$$\text{P\%} = \frac{x \times 18}{144 \times x} \times 100 = 12\frac{1}{2}\%$$

5. Let the C.P. of 25 chairs be x

Then C.P. of 1 chair = $\frac{x}{25}$

But, S.P. of 30 chairs = x

S.P. of 1 chair = $\frac{x}{30}$

$$\text{Loss} = \frac{x}{25} - \frac{x}{30} = \frac{x}{150}$$

$$\text{Loss\%} = \frac{x \times 25}{150 \times x} \times 100 = \frac{50}{3} = 16\frac{2}{3}\%$$

6. As vendor brought orange at 20 for ₹ 56

C.P. of 1 orange = $\frac{56}{20}$

= ₹2.8

S.P. of 12 oranges = ₹35

$$\text{S.P. of 1 orange} = \frac{35}{12}$$

$$\text{Profit} = \frac{35}{12} - \frac{28}{10} = \frac{350-336}{12\times10} = \frac{14}{120} = \frac{7}{60}$$

$$\text{Profit \%} = \frac{7\times10\times100}{60\times28} = \frac{25}{6} = 4\frac{1}{6}\%$$

7. ₹400 = S.P. of article

   Let the C.P. be x

$$\text{Profit is to be} = \frac{1}{4}x$$

$$g\% = \frac{1\times x}{4\times x}\times100 = 25\%$$

8. Dress sold at = ₹960

   Let the C.P. of = x

   The gain amount $= \frac{1}{9}x$

   G = S.P. – C.P. as,

$$\frac{1}{9}x = 960 - x$$

$$\frac{1\times x}{9} + x = 960$$

$$10x = 960\times9$$

$$x = 864$$

$$\text{C.P.} = ₹864$$

$$\text{Profit\%} = \frac{1\times x}{9\times x}\times100 = 11.11\%$$

9. C.P. = ₹3,00,000

   S.P. of $\frac{1}{3}$ is at loss of 20%

$$\text{C.P. of } \frac{1}{3} = \frac{1}{3}\times300000 = 1,00000$$

$$\text{Then 20\% of it} = \frac{20}{100}\times100000 = 20000$$

$$\text{Loss} = 20000$$

S.P. of $\frac{2}{5}$ is at gain 25%

$$\frac{2}{5} \times 300000 = 120000$$

Gain = $\frac{25}{100} \times 120000 = 30000$

Overall 10% is gain then = $\frac{10}{100} \times 300000 =$ ₹30000

And net gain $= 30000 - 20000 = 10000$

So, we should have a gain $= 30000 - 10000 = 20000$

As the land remaining is $= 1 - \frac{1}{3} - \frac{2}{5} = \frac{15-5-6}{15} = \frac{4}{15}$

C.P. of $\frac{4}{15} \times 300000 = 80000$

The S.P. would be $= 80000 + 20000 =$ ₹100000

10. Let the C.P. of 1000 gm gold be x

Then C.P. of 1 gm $= \frac{x}{1000}$

The S.P. of 980 gm $= x$

$$\text{S.P. of 1 gm gold} = \frac{x}{980}$$

$$\text{Profit} = \frac{x}{980} - \frac{x}{1000}$$

$$= \frac{20x}{980 \times 1000} = \frac{x}{49000}$$

$$P\% = \frac{x \times 1000}{49000 \times x} \times 100 = 2\frac{2}{49}\%$$

11. C.P. of two fans ₹ 2160

Let the C.P. of 1st fan be x

C.P. of 2nd fan will be = ₹ (2160 – x)

Profit on 1st fan = 15% of x

$$= \frac{15x}{100}$$

$$\text{S.P. of 1st fan} = ₹\left(x + \frac{15x}{100}\right)$$

$$\text{Loss on 2nd fan} = 9\% \text{ of } ₹\,(2160 - x)$$

$$= ₹\left(\frac{9}{100} \times (2160 - x)\right)$$

$$\text{S.P. of 2nd fan} = \text{C.P.} - \text{Loss}$$

$$= ₹\,(2160 - x) - \frac{9(2160 - x)}{100}$$

$$= (2160 - x)\left(\frac{91}{100}\right)$$

It is given that S.P. of each fan is the same.

$$\frac{x}{1} + \frac{15x}{100} + (2160 - x)\left(\frac{91}{100}\right) = 2160$$

$$\frac{115x}{100} + \frac{91(2160 \times x)}{100} = 2160$$

$$11x + 19650 - 91x = 216000$$

$$24x = 19440$$

$$x = \frac{19440}{24} = 810$$

$$\text{and cost of 2nd fan} = 2160 - 810$$

$$= ₹\,1350$$

12. Let C.P. of bucket be = ₹x

Loss % = 8%

So, $\text{S.P.} = x - \left(\frac{x \times 8}{100}\right)$

$$= \frac{92x}{100}$$

No, again profit = 8%

So, $\text{S.P.} = x + \left(\frac{x \times 8}{100}\right)$

$$= \frac{108x}{100}$$

According to question

$$\frac{108x}{100} - \frac{92x}{100} = 28$$

$$108x - 92x = 2800$$

$$16x = 2800$$

$$x = 175$$

So, C.P. of the basket = ₹175

13. Cost of 50 chairs = ₹50,000

C.P. of 1 chair = ₹50000 ÷ 50

= ₹1000

So, S.P. of one damage chair = $₹1000 \times \frac{3}{4}$

S.P. of 20 damage chairs = ₹750 × 20 = ₹ 1500

To gain 35% overall profit, we should sell the chairs in

$$\left(50000 + 50000 \times \frac{35}{100}\right) = 67500$$

So, S.P. of 30 chairs = ₹67500 – ₹15000 = ₹52500

So, S.P. of 1 chair = ₹ $\frac{52500}{30}$ = ₹1750 each

14. Let the actual C.P. be = ₹x

S.P. in first case $= x + \left(x \times \frac{5}{100}\right)$

$$= \frac{x}{1} + \frac{x}{20} = \frac{21x}{20}$$

S.P. in second case $= x - \left(x \times \frac{5}{100}\right)$

$$= \frac{19x}{20}$$

According to question :

$$\frac{21x}{20} - 50 = \frac{19x}{20}$$

$$\frac{21x - 1000 = 19x}{20}$$

$$21x - 19x = 1000$$

$$2x = 1000$$

$$x = 500$$

So, $\text{S.P.} = \frac{21 \times 500}{20} = ₹525$

15. Let C.P. be = 100x

5% of 100 x = 5x

So, First S.P. = 105x

New C.P. = 95x

10% of 95x = 9.5x

New S.P. = 95x + 9.5x

= 104.5 x

According to question

$$105x - 104.5x = 2$$

$$0.5x = 2$$

$$x = \frac{2}{5} \times 10 = 4$$

So, C.P. will be = 4 × 100 = 400

Exercise 8C

1. (i) MP = ₹650 and discount = 10%

$$\text{Discount\%} = \frac{D}{\text{M.P.}} \times 100$$

$$\frac{650 \times 10}{100} = D$$

$$D = ₹65$$

$$\text{S.P.} = (\text{M.P.} - D)$$

$$= (650 - 65)$$

$$= ₹585$$

(ii) MP = ₹ 5450 and discount = 5%

$$D\% = \frac{D}{M.P.} \times 100$$

$$\frac{5 \times 5450}{100} = D$$

$$D = ₹272.5$$

$$S.P. = (MP - D)$$

$$= (5450 - 272.5)$$

$$= ₹5177.5$$

2. (i) S.P. = ₹3430 ; D = 2%

Let M.P. = ₹100

Discount = ₹2

S.P. = ₹(100 – 2) = ₹98

So, Actual M.P. $= \frac{100}{98} \times \frac{3430}{1} = ₹3500$

(ii) S.P. = ₹9250; D = 7.5%

Let M.P. = ₹ 100

Discount = ₹ 7.50

S.P. = ₹(100 – 7.50) = ₹92.50

So, Actual M.P. $= \frac{100}{92.50} \times \frac{9250}{1}$

= ₹ 10,000

3. (i) $D\% = \frac{M.P. - S.P.}{M.P.} \times 100$

$$D\% = \frac{625 - 562.5}{62.5} \times 100$$

$$= \frac{6250.0}{625} = 10\%$$

(ii) $D\% = \frac{M.P. - S.P.}{M.P.} \times 100$

$$D\% = \frac{1600 - 1180}{1600} \times 100 = \frac{420 \times 100}{1600}$$

$$D\% = 26\frac{1}{4}\%$$

4. M.P. = ₹18500, D% = 12%

$$D\% = \frac{\text{Discount}}{\text{M.P.}} \times 100$$

$$\frac{12 \times 18500}{100} = \text{Discount}$$

$$2220 = \text{Discount}$$

So, $\text{S.P.} = \text{M.P.} - \text{Discount}$

$= 18500 - 2220$

$= ₹\,16280$

5. Let M.P. = ₹100; D = 12% or ₹ 12

So, $\text{S.P.} = ₹100 - ₹12$

$= ₹88$

So, $\text{M.P.} = \frac{100}{88} \times \frac{880}{1} = ₹\,1000$

6. C.P. = 3000; D = 10%; P = 20%

$$\text{S.P.} = \frac{CP(100+P)}{100}$$

$$= \frac{3000(100+20)}{100}$$

$$= \frac{3000 \times 120}{100}$$

$$= ₹\,3600$$

Let, M.P. = ₹100; D = 10%

So, $\text{S.P.} = ₹\,100 - ₹\,10 = ₹\,90$

So, $\text{Actual M.P.} = \frac{100}{90} \times \frac{3600}{1}$

$= ₹\,4000$

7. Let C.P. = ₹100; Profit % = 20; Profit = ₹20

$$\text{S.P.} = ₹100 + ₹20 = ₹120$$

$$\text{Actual profit} = ₹360$$

$$\text{Actual S.P.} = \frac{120}{20} \times 360 = ₹2160$$

Now, Let M.P. be = ₹100

Discount = 25%

So, S.P. = ₹75

$$\text{Actual M.P.} = \frac{100}{75} \times \frac{2160}{1} = ₹2880$$

8. Let the C.P. be ₹ 100; The gain required = 20%

The S.P. = ₹ (100 + 20) = ₹ 120

Discount allowed = 25%

Let the marked price be x

Then discount = 25% of x = ₹ $\left(\frac{25}{100} \times x\right) = \frac{x}{4}$

$$\text{S.P.} = \text{M.P.} - \text{Discount}$$

$$= ₹\left(x - \frac{x}{4}\right) = \frac{3x}{4}$$

But S.P. = ₹ 120

$$\frac{3x}{4} = 120$$

$$x = \frac{120 \times 4}{3}$$

$$x = ₹160$$

Hence the shopkeeper should mark 60% more.

9. The C.P. = ₹450, The gain required = 20%

$$\frac{20}{100} \times 450 = 90$$

$$\text{S.P.} = ₹(450 + 90) = 540$$

Discount = 10%

Let S.P. be $= ₹x$

Discount $= 10\%$ of x

$= \frac{x}{10}$

So, S.P. $= x - \frac{x}{10} = \frac{9x}{10} = 540$

$x = \frac{540 \times 10}{9}$

$= ₹600$

10. M.P. $= ₹18500$

Ist Discount $= \frac{20}{100} \times 18500$

$= ₹3700$

$= 18500 - 3700 = ₹14800$

IInd Discount $= 5\%$ of 14800

$= \frac{5}{100} \times 14800 = 740$

Net Selling price $= (14800 - 740)$

$= ₹14060$

11. Let the marked price be ₹ 100

Then the Ist discount given on it $= ₹20$

The price after discount $= ₹(100 - 20) = ₹80$

The next discount $= 5\%$ of 80

$= \frac{5}{100} \times 80 = ₹4$

The price after the second discount $= ₹(80 - 4) = ₹76$

The net S.P. $= ₹76$

The single discount equivalent to

Discount $= (100 - 76) = 24\%$

Check Your Mental Math IQ

1. $$\frac{20}{100} \times x = 200$$
$$x = \frac{200 \times 10}{2}$$
$$x = 1000$$

2. $$100 = \frac{400 \times x}{100}$$
$$25\% = x$$

3. After 1 year salary will be increase $= \frac{10}{100} \times 500$

So it become $=$ ₹5500

After 2 years $= \frac{10}{100} \times 5500$

i.e. $= 5500 + 550 =$ ₹6050

4. Total C.P. $= 5000 + 250 = 5250$

S.P. $=$ ₹4000

Loss $= 5250 - 4000 =$ ₹1250

5. $$P = \frac{20}{100} \times 3000$$
$$P = 600$$
S.P. $= 3000 + 600 =$ ₹3600

6. $$\text{M.P.} = \frac{100 \times S.P.}{100 - \text{Discount}}$$

D% = 12%; S.P. = ₹880

$$\text{M.P.} = \frac{100 \times 880}{(100 - 12)} \times 3000$$
$$= \frac{100 \times 880}{88}$$
M.P. $=$ ₹1000

7. C.P. of 6 article ₹ 5

C.P. of 1 article = ₹ $\frac{5}{6}$

S.P. of 5 article = ₹6

S.P. of 1 article = ₹$\frac{6}{5}$

$$\text{Gain} = \frac{6}{5} - \frac{5}{6} = \frac{36-25}{30} = \frac{11}{30}$$

$$\text{Gain\%} = \frac{11 \times 6}{30 \times 5} \times 100 = 44\%$$

Multiple Choice Questions (M.C.Q.)

1. $$50 \times ? = \frac{20}{100} \times 4000$$

$$? = \frac{800}{50} = 16 \quad \text{(b)}$$

2. Let the original number be x

as one increased 20% = $\frac{20}{100} \times x = \frac{x}{5}$

$$\text{New one is} = x + \frac{x}{5} = \frac{6x}{5}$$

Then Increased 20% = $\frac{20 \times 6x}{100 \times 5} = \frac{6x}{25}$

$$\text{Newer one will be} = \frac{6x}{25} + \frac{6x}{5} = \frac{6x + 30x}{25} = \frac{36x}{25}$$

So, total increased = $\frac{36x}{25} - x = \frac{11x}{25}$

$$\text{\% increased} = \frac{11x}{25 \times x} \times 100 = 44\%(d)$$

3. $$\text{C.P.} = \frac{S.P. \times 100}{(100 + \text{P\%})}$$

$$\text{C.P.} = \frac{420 \times 100}{(100+5)} = \frac{420 \times 100}{105}$$

= ₹ 400 (a)

4. First chair = ₹600; Gain = 20%

$$\text{First C.P.} = \frac{S.P. \times 100}{(100 + \text{P\%})}$$

$$= \frac{600 \times 100}{120} = ₹500$$

$$\text{Second C.P.} = \frac{600 \times 100}{75} = ₹800$$

$$\text{Total C.P.} = ₹500 + ₹800$$

$$= ₹1300$$

$$\text{Total S.P.} = ₹600 + ₹600 = ₹1200$$

$$= ₹1300 - ₹1200$$

$$= ₹100 \quad \text{(c)}$$

5. Let C.P. be x

$$\text{S.P. of article} = \frac{3}{5} \times x$$

$$\text{L} = \text{C.P.} - \text{S.P.}$$

$$\text{L} = x - \frac{3}{5} \times x$$

$$= \frac{2x}{5}$$

$$\text{L\%} = \frac{2x}{5 \times x} \times 100$$

$$\text{L} = 40\% \quad \text{(a)}$$

6. S.P. = ₹4200; M.P. = ₹5000;

$$\text{D} = \text{M.P.} - \text{S.P.}$$

$$= 5000 - 4200 = ₹800$$

$$\text{D\%} = \frac{800}{5000} \times 100$$

$$\text{D\%} = 16\% \quad \text{(a)}$$

9 Compound Interest

1. $$\text{Amount} = P\left(1+\frac{r}{100}\right)^t$$
$$= 2000\left(1+\frac{15}{100}\right)^3$$
$$= 2000\times\frac{115}{100}\times\frac{115}{100}\times\frac{115}{100}$$
$$= ₹3041.75$$

So, compound interest = Amount – Principal
= ₹ 3041.75 – ₹ 2000 = ₹ 1041.75

2. P = ₹ 6000; R = 12%; T = 2 years

$$\text{Amount} = 6000\left(1+\frac{12}{100}\right)^2$$
$$= 6000\times\frac{112}{100}\times\frac{112}{100} = ₹7526.40$$

Compound Interest = ₹7526.40 – ₹6000 = ₹ 1526.40

3. Here, P = ₹50000, T = 3 years; Rate 10%

$$\text{Amount} = 50000\left(1+\frac{10}{100}\right)^3$$
$$= 50000\times\frac{110}{100}\times\frac{110}{100}\times\frac{110}{100} = ₹66550$$

Compound interest = ₹ 66550 – ₹ 50000 = ₹ 16550

4. Here, P = ₹20000; Time = 3 years; Rate = 5%

$$\text{Amount} = 20000\left(1+\frac{5}{100}\right)^3$$

$$= 20000 \times \frac{105}{100} \times \frac{105}{100} \times \frac{105}{100}$$

$$= ₹23150.50$$

5. P = ₹ 25000; Time = $1\frac{1}{2}$; years; Rate = 8% half yearly

As interest compound half yearly

So, Time = $1\frac{1}{2} \times 2$ = 3 half years

And Rate = $\frac{8}{2} = 4\%$

So, Amount = $25000\left(1+\frac{4}{100}\right)^3$

$$= 25000 \times \frac{26}{25} \times \frac{26}{25} \times \frac{26}{25} = ₹28121.60$$

So, Compound Int. = ₹ 28121.60 – ₹25000 = ₹ 3121.60

6. Here, P = ₹2000, Time = $1\frac{1}{2}$ years ; R = 10%

As interest compounded half yearly

So, Time will be = 3 years and Rate 5%

So, Amount = $2000\left(1+\frac{5}{100}\right)^3$

$$= 2000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} = ₹2315.25$$

So, CI = ₹2315.25 – ₹2000 = ₹315.25

7. Here P = ₹25000, Time = $1\frac{1}{2}$ years; Rate = 20%

As interest compounded half yearly

So, Time will be = 3 years and Rate = 10%

So, Amount = $25000\left(1+\frac{10}{100}\right)^3$

$$= 25000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10} = ₹33275$$

8. Here P = ₹10000; Time = 6 months; Rate = 12%

As interest compounded quarterly

So, Time will be = 2 quarters; Rate = $\frac{12}{4} = 3\%$

So, $$\text{Amount} = 10000\left(1+\frac{3}{100}\right)^2$$

$$= 10000 \times \frac{103}{100} \times \frac{103}{100} \times \frac{103}{100} = ₹10609$$

Exercise 9B

1. P = ₹8500; R = 8%; T = 2 years

$$A = P\left(1+\frac{R}{100}\right)^T$$

$$= 8500\left(1+\frac{8}{100}\right)^2$$

$$= 8500 \times \frac{108}{100} \times \frac{108}{100} = ₹9914.40$$

$$\text{C.I.} = 9914.40 - 8500 = ₹\,1414.40$$

2. P = ₹50000; R = 10%; T = $2\frac{1}{2}$ years

$$A = 50000\left(1+\frac{10}{100}\right)^2 \times \left(1+\frac{10}{100} \times \frac{1}{2}\right)^1$$

$$= 50000 \times \frac{110}{100} \times \frac{110}{100} \times \frac{21}{20} = ₹635250$$

$$\text{C.I.} = ₹\,635250 - ₹\,50000 = ₹\,135250$$

3. P = ₹ 2500; R = 9%; T = 2 years

$$A = 2500\left(1+\frac{9}{100}\right)^2$$

$$= 25000 \times \frac{109}{100} \times \frac{109}{100} = ₹2970.25$$

$$\text{C.I.} = ₹(2970.25 - 2500) = ₹\,470.25$$

4. P = ₹ 3200; R = 25%; T = 3

$$A = 3200\left(1+\frac{25}{100}\right)^3$$

$$= 3200 \times \frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} = ₹6250$$

$$\text{C.I.} = ₹(6250 - 3200) = ₹3050$$

5. P = ₹ 15625; R = 4%; T = 2 years

$$A = 15625\left(1+\frac{4}{100}\right)^2$$

$$= 15625 \times \frac{104}{100} \times \frac{104}{100} = ₹16900$$

$$\text{C.I.} = ₹16900 - ₹15325 = ₹1275$$

6. P = ₹6400; R = 17.5%; T = 2 years

$$A = 6400\left(1+\frac{17.5}{100}\right)^2$$

$$= 6400 \times \frac{117.5}{100} \times \frac{117.5}{100} = ₹8836$$

$$\text{C.I.} = ₹(8836 - 6400) = ₹2436$$

7. P = ₹1000; T = 2 years; R = 20%
   As. C.I. compounded half yearly
   So, T = 4 half years; R = 10%

$$A = 1000\left(1+\frac{10}{100}\right)^4$$

$$= 1000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10} = ₹1464.10$$

$$\text{C.I.} = ₹(1464.10 - 1000) = ₹\,464.10$$

8. P = ₹4016; T = 18 months; R = 12.5%

As, CI compounded half yearly

So, Time = 3 half years; $R = \frac{12.5}{2} = 6.25\%$

$$A = 4016\left(1+\frac{6.25}{100}\right)^3$$

$$= 4016 \times \frac{106.25}{100} \times \frac{106.25}{100} \times \frac{106.25}{100} = ₹4817.04$$

9. Let the principal be = ₹100

According to question,

$$\text{Interest} = \frac{100 \times 3 \times 6.25}{100} = ₹18.75$$

$$\text{Actual principal} = \frac{100}{18.75} \times 2400 = ₹12800$$

Now, P = ₹12800, Time = 3 years; Rate = 6.25%

$$A = 12800\left(1+\frac{6.25}{100}\right)^3$$

$$= 12800 \times \frac{106.25}{100} \times \frac{106.25}{100} \times \frac{106.25}{100}$$

$$= ₹\,15353.13$$

$$\text{C.I.} = ₹(15353.13 - 12800) = ₹2553.13$$

10. P = ₹320000; T = 1 year; R = 20%

As, CI is compound quarterly

So, T = 4 quarters, $R = \frac{20}{4} = 5\%$

$$A = 320000\left(1+\frac{5}{100}\right)^4$$

$$= 320000 \times \frac{105}{100} \times \frac{105}{100} \times \frac{105}{100} \times \frac{105}{100}$$

$$= ₹\,388962$$

So, $\text{CI} = ₹(388962 - 320000) = ₹\,68962$

11. P = ₹ 80000; Time = 3 years; R = 4%, 5%. 10%

$$A = 80000\left(1+\frac{4}{100}\right)\times\left(1+\frac{5}{100}\right)\times\left(1+\frac{10}{100}\right)$$

$$= 80000\times\frac{104}{100}\times\frac{105}{100}\times\frac{110}{100} = ₹96096$$

$$\text{C.I.} = ₹(96096-80000) = ₹16096$$

12. P = ₹8000; Time = 2 years; Rate = 5%, 15%

$$A = 8000\left(1+\frac{5}{100}\right)\times\left(1+\frac{15}{100}\right)$$

$$= 8000\times\frac{105}{100}\times\frac{115}{100} = ₹9660$$

13. P = ₹ 31250; T = 2 years 3 months or $= 2\frac{1}{4}$ years; R = 16%

$$A = 31250\left(1+\frac{16}{100}\right)^2\times\left(1+\frac{16}{100}\times\frac{1}{4}\right)$$

$$= 31250\times\frac{116}{100}\times\frac{116}{100}\times\frac{104}{100} = ₹43732$$

14. P = ₹ 24000; T = 9 months; R = 20p/100p or 20%

As the CI compounded quarterly

So, Time = 3 quarters; $R = \frac{20}{4} = 5\%$

$$A = 24000\left(1+\frac{5}{100}\right)^3$$

$$= 24000\times\frac{105}{100}\times\frac{105}{100}\times\frac{105}{100} = ₹27783$$

$$\text{C.I.} = ₹(27783-24000) = ₹3783$$

15. Let the principal be = ₹ 100; Time = 4 years; Rate 4%

$$\text{Interest} = \frac{100\times4\times4}{100} = ₹16$$

So, Actual principal $= \frac{100}{16}\times1600 = ₹10000$

Now, Time = 4 years; Rate = 10%

$$A = 10000\left(1+\frac{10}{100}\right)^4$$

$$= 10000\times\frac{110}{100}\times\frac{110}{100}\times\frac{110}{100}\times\frac{110}{100}$$

$$= ₹\,14641$$

So, C.I. = ₹(14641 – 10000) = ₹4641

Exercise 9C

1. Let the principal be = ₹100; Rate = 15%; Time = 2 years

$$\text{Amount} = 100\left(1+\frac{15}{100}\right)^2$$

$$= 100\times\frac{115}{100}\times\frac{115}{100} = ₹132.25$$

C.I. = ₹(132.25 – 100) = ₹32.25

If compound interest is ₹ 32.25 then principal be = ₹100

If compound interest is ₹ 1 than principal be = $\frac{100\times100}{3225}$

If compound interest is ₹ 1290 then principal be

$$= \frac{100\times100\times1290}{3225} = ₹4000$$

2. Let the principal be = ₹ 100; Time = 2 years; Rate = 5%

$$\text{Amount} = 100\left(1+\frac{5}{100}\right)^2$$

$$= 100\times\frac{105}{100}\times\frac{105}{100} = ₹110.25$$

Actual Principal will be = $\frac{100\times100}{11025}\times\frac{55125}{1} = ₹50000$

3. Principal = ₹ 10000; Time = 3 years; Rate = x%

Amount = 13310

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^T$$

$$13310 = 10000\left(1+\frac{R}{100}\right)^3$$

$$\frac{13310}{10000} = 10000\left(1+\frac{R}{100}\right)^3$$

$$\left(\frac{11}{10}\right)^3 = 10000\left(1+\frac{R}{100}\right)^3$$

$$\frac{11}{10} = 1+\frac{R}{100} \Rightarrow \frac{R}{100} = \frac{11}{10}-\frac{1}{1} = \frac{1}{10} \Rightarrow R = 10\%$$

4. R = x%; Time = $1\frac{1}{2}$ years or 3 half years; Principal = ₹ 800

   Amount = ₹ 926.10

   As interest compound half yearly, so rate will be = $\frac{x}{2}\%$

$$\frac{92610}{100} = 800\left(1+\frac{x}{2\times 100}\right)^3$$

$$\frac{92610}{80000} = \left(1+\frac{x}{200}\right)^3$$

$$\left(\frac{21}{20}\right)^3 = \left(1+\frac{x}{200}\right)^3$$

$$1+\frac{x}{200} = \frac{21}{20}$$

$$\frac{x}{200} = \frac{21}{20}-\frac{1}{1} = \frac{1}{20}$$

$$x = \frac{1}{20}\times 200 = 10\%$$

   So, Rate = 10%

5. Time = x years; Rate = 5%; Principal = ₹800; Amount = ₹882

$$882 = 800\left(1+\frac{5}{100}\right)^x$$

$$\frac{882}{800} = \left(\frac{105}{100}\right)^x$$

$$\frac{441}{400} = \left(\frac{21}{20}\right)^x$$

$$\left(\frac{21}{20}\right)^3 = \left(\frac{21}{20}\right)^2$$

$$x = 2$$

6. Time = x years; Principal = ₹ 64000; Rate = 5%
   Amount = ₹ 68921
   As interest compounded half yearly
   So time will be = 2x and Rate will be = $\frac{5}{2}\%$

$$68921 = 64000\left(1+\frac{5}{2\times 100}\right)^{2x}$$

$$\frac{68921}{64000} = \left(\frac{41}{40}\right)^{2x}$$

$$\left(\frac{41}{40}\right)^3 = \left(\frac{41}{40}\right)^{2x}$$

$$2x = 3$$

$$x = \frac{3}{2} = 1\frac{1}{2} \text{ years}$$

7. Principal = ₹ 1800; Rate = 10%; Time = x years
   Compund interest = ₹ 378

$$\text{Amount} = ₹(1800+378) = ₹\,2178$$

$$\text{Amount} = \text{Principal}\left(1+\frac{R}{100}\right)^T$$

$$2178 = 1800\left(1+\frac{10}{100}\right)^x$$

$$\frac{2728}{1800} = \left(\frac{11}{10}\right)^x$$

$$\frac{121}{100} = \left(\frac{11}{10}\right)^x$$

$$\left(\frac{11}{10}\right)^2 = \left(\frac{11}{10}\right)^x$$

$$x = 2$$

So, Time = 2 years

8. Let the principal be = ₹ 100; Rate == 5%; Time = 3 years

$$\text{Simple interest} = \frac{100 \times 5 \times 3}{100} = ₹15$$

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^T$$

$$A = 100\left(1 + \frac{5}{100}\right)^3$$

$$= 100 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} = \frac{9261}{80}$$

$$\text{Compound Interest} = \frac{9261}{20} - \frac{100}{1} = \frac{9261 - 8000}{80} = \frac{1261}{80}$$

Difference between compound interest and simple interest

$$= \frac{1261}{80} - \frac{15}{1} = \frac{1261 - 1200}{80} = ₹\frac{61}{80}$$

$$\text{Actual principal will be} = \frac{100 \times 80 \times 61}{61 \times 1} = ₹8000$$

9. Let the principal be = ₹ 100; Time = 3 years; Rate = $6\frac{2}{3} = \frac{20}{3}\%$

$$\text{Simple interest} = \frac{100 \times 3 \times 20}{3 \times 100} = ₹20$$

$$\text{Amount} = 100\left(1 + \frac{20}{3 \times 100}\right)^3$$

$$= 100 \times \frac{16}{15} \times \frac{16}{15} \times \frac{16}{15} = \frac{409600}{3375}$$

$$\text{C.I.} = \frac{409600}{3375} - \frac{100}{1} = \frac{409600 - 337500}{3375}$$

$$= ₹\ \frac{72100}{3375}$$

Difference between C.I. and S.I.

$$= \frac{72100}{3375} - \frac{20}{1} = \frac{72100 - 67500}{3375} = \frac{4600}{3375}$$

Actual principal will be $= \frac{100 \times 3375}{4600} \times 184 = ₹13500$

10. Amount after 2 years = ₹ 2205

Amount after 3 years = ₹2315.25

Difference = ₹(2315.25 – 2205) = ₹110.25

This difference is interest on ₹ 2205 for 1 year

So Rate $= \frac{11025}{100} \times \frac{100}{2205} = 5\%$

Exercise 9D

1. Present population = 80000; Time = 2 years; Rate = 7.5%

Population after 2 years $= 80000\left(1 + \frac{7.5}{100}\right)^2$

$$= 80000 \times \frac{107.5}{100} \times \frac{107.5}{100} = 92450$$

2. Present population = 40000; Rate of birth = 4%;

Rate of death = 2%

Nett Rate of Birth = 4 – 2 = 2%

Population after 2 years $= 40000\left(1 + \frac{2}{100}\right)^2$

$$= 40000 \times \frac{51}{50} \times \frac{51}{50} = 41616$$

3. Population before 3 years = 160000; Rate of increse = 3%, 2.5%, 5%

Present population $= 16000\left(1 + \frac{3}{100}\right)\left(1 + \frac{2.5}{100}\right)\left(1 + \frac{5}{100}\right)$

$$= 160000 \times \frac{103}{100} \times \frac{102.5}{100} \times \frac{105}{100} = 177366$$

4. Population before 2 years = 62500; Rate of decrease = 4%

$$\text{Present population} = 62500\left(1 - \frac{4}{100}\right)^2$$

$$= 625000 \times \frac{24}{25} \times \frac{24}{25} = 57600$$

5. Present value of refrigerator = ₹ 9680; Time = 2 years;
   Rate of depreciation = 12%
   Let the cost before 2 years be = x
   So,

$$9680 = x\left(1 - \frac{12}{100}\right)^2$$

$$9680 = x \times \frac{88}{100} \times \frac{88}{100}$$

$$x = \frac{9680 \times 100 \times 100}{88 \times 88} = ₹12500$$

6. Cost of new car = ₹ 360000; Rate of depreciation = 10% and 20%

$$\text{Cost of car after 3 years} = 360000\left(1 - \frac{10}{100}\right)^2 \times \left(1 - \frac{20}{100}\right)$$

$$360000 \times \frac{9}{10} \times \frac{9}{10} \times \frac{4}{5} = ₹233280$$

Check Your Mental Maths (I.Q.)

1. Compound interest
2. Principal
3. ₹ 630.50
4. 97.20
5. ₹ 18300
6. 10%

Multiple Choice Questions (M.C.Q.)

1. (b)
2. (c)
3. (c)
4. (b)
5. (b)
6. (b)

10 Direct and Inverse Variation

Exercise 10A

1. (i)

a	4	7	21	28
b	12	21	63	84

$\frac{4}{12}=\frac{1}{3}$, $\frac{7}{21}=\frac{1}{3}$, $\frac{21}{63}=\frac{1}{3}$, $\frac{28}{84}=\frac{1}{3}$

as direct variation.

(ii)

a	1.25	2.5	5	7.5
b	5	10	20	30

$\frac{1.25}{5}=.25$, $\frac{2.5}{10}=.25$, $\frac{5}{25}=.25$, $\frac{7.5}{30}=.25$

as direct vari

(iii)

a	1	2	3	4
b	2	1	6	3

as $\frac{1}{2}\neq\frac{2}{1}\neq\frac{3}{6}\neq\frac{4}{3}$ indirect variation

2. (i)

x	2.5	a	b	c	17
y	5	8	18	30	d

$\frac{2.5}{5}=.5$,

$\frac{a}{8}=.5$, a = 4 $\frac{b}{18}=.5$ b = 9

$\frac{c}{30}=.5$, c = 15 $\frac{17}{d}=.5$ $d=\frac{17}{.5}=34$

(ii)

x	3	4	b	25	d
y	a	48	84	c	1860

$\frac{4}{48}=\frac{1}{12}$

$$\frac{3}{a}=\frac{1}{12},\ a=36 \qquad \frac{b}{84}=\frac{1}{12} \quad b=\frac{84}{12}=7$$

$$\frac{25}{c}=\frac{1}{12},\ c=300 \qquad \frac{d}{1860}=\frac{1}{12} \quad d=\frac{1860}{12}=155$$

3. If 20 men assemble, 8 machines

Men	20	—
Machine	8	12

$$\frac{20}{8}=\frac{a}{12}$$

$$\frac{20\times 12}{8}=a$$

a = 30

So, 30 men are required.

4.

Hours	5	20
Tools	120	a

$$\frac{5}{120}=\frac{20}{a}$$

$$a=\frac{20\times 120}{5}$$

a = 480

5.

Steps	150	360
Distance	125	b

$$\frac{150}{125}=\frac{300}{b}$$

$$b=\frac{360\times 125}{150}$$

b = 300 steps it moves

6.

Copies	136	c
Self length	3.4	5.1

$$\frac{136}{3.4}=\frac{c}{5.1}$$

$$c = \frac{5.1 \times 136}{3.4}$$

c = 204 copies we required

7.

Meter	40	a
Cost	1940	727.50

$$\frac{40}{1940} = \frac{a}{727.50}$$

$$a = \frac{4 \times 7275}{1940}$$

a = 15 m

So, 15 m of cloth is required.

8.

Charge	520	559
Journey	200	a

$$\frac{520}{200} = \frac{559}{a}$$

$$a = \frac{559 \times 200}{520}$$

a = 215 km

at ₹ 529 it moves 215 km

9.

Thickness	35	c
Number	12	294

$$\frac{35}{12} = \frac{c}{294}$$

$$\frac{35 \times 294}{12} = c$$

c = 857.5 mm

So, 294 cardboard has 857.5 mm thickness

10.

Weight	150	a
expansion	2.9	17.4

$$\frac{150}{2.9} = \frac{a}{17.4}$$

$$a = \frac{150 \times 17.4}{1.9}$$

$a = 900$ gm

Exercise 10B

1. (i)

x	9	18	2	12
y	8	4	30	6

$9 : 18 = 4 : 8, \quad \frac{1}{2} = \frac{1}{2}$

But $2 : 12 \neq 6 : 30, \quad \frac{1}{6} \neq \frac{1}{5}$ No.

(ii)

x	2	16	8	4
y	40	5	10	20

$2:16 = \frac{1}{8} \qquad 5:40 = \frac{1}{8}$

$16 : 8 = 2 = 10 : 5 = 2$

$8 : 4 = 2 = 20 : 10 = 2$ Yes.

(iii)

x	8	16	32	256
y	32	16	8	1

$8 : 16 = \frac{1}{2}, \quad 16 : 32 = \frac{1}{2}, \quad 32 : 256 = \frac{1}{8}$

$16 : 32 = \frac{1}{2}, \quad 8 : 16 = \frac{1}{2} \qquad 1 : 8 = \frac{1}{8}$

Yes, True

2. (i)

a	8	2	b	5	1
b	10	a	20	c	80

$8 : 2 = a : 10 \qquad 40 = a$

$2 : b = 20 : 40$

$\frac{2}{b} = \frac{20}{40} \Rightarrow b = 4$

$4 : 5 = c : 20 \Rightarrow c = 16$

(ii)

a	16	32	8	128
b	4	a	b	0.5

$16 : 32 = a : 4$

$\frac{1}{2} = \frac{a}{4}$

$a = 2$

$32 : 8 = b : 2$

$\frac{32}{8} = \frac{b}{2}$

$b = 8$

3. Total expenditure = 52 × 1050 = ₹54600

New cost = 1050 + 42 = 1092

Cycle	52	x
Cost	1050	1092

$$\frac{52}{x} = \frac{1092}{1050}$$

$$\frac{52 \times 1050}{1092} = x$$

$x = 50$

So 50 cycle is to be purchased now.

4.

Men	56	a
Days	42	14

$$\frac{56}{a} = \frac{14}{42}$$

$a = 168$

So, 168 men do work in 14 days.

5.

Speed	18	a
Time	30	20

$$\frac{18}{a} = \frac{20}{30}$$

$$\frac{18 \times 30}{20} = a$$

$a = 27$ km/hour

6.

Person	1800	x
Days	40	24

$$\frac{1800}{x} = \frac{24}{40}$$

$$\frac{1800 \times 40}{24} = x$$

$$x = 3000$$

So, 3000 persons required to to work in 24 days

7.

Soldiers	1000	a
Days	20	25

$$\frac{1000}{a} = \frac{25}{20}$$

$$\frac{1000 \times 20}{25} = a$$

$$a = 800$$

For 800 soldiers food lasted for 25 days

So, $1000 - 800 = 200$

So, 200 soldiers will transfer.

8. 120 men has food provision for 200 days

After 5 days $(200 - 5) = 195$

As 30 died, so $= 120 - 30 = 90$ left

Men	120	90
Days	195	a

$$\frac{120}{90} = \frac{a}{195}$$

$$\frac{120 \times 195}{90} = a$$

$$a = 260$$

So, 260 days

9. 60 days for 500 students

Days left $= 60 - 12 = 48$

as $500 + 300 = 800$ students

Students	500	800
Days	48	a

$$\frac{500}{800} = \frac{a}{48}$$

$$\frac{500 \times 48}{800} = a$$

a = 30 days

10. (i) x = 6 when y = 16 find

x	6	8
y	16	?

$$\frac{6}{8} = \frac{y}{16}$$

12 = y

(ii) x = 10 when y = 30, y = 24

x	10	?
y	30	24

$$\frac{10}{y} = \frac{24}{30}$$

$$\frac{300}{24} = a$$

y = 12.5

Check Your Mental Maths I.Q.

1. Inverse proportion
2. Direct proportion
3. Direct proportion
4. Inverse proportion
5.

Meter	5	70
Shirt	2	a

$$\frac{5}{2} = \frac{70}{a}$$

$$a = \frac{70 \times 2}{5} = 28\text{shirts}$$

We

6.

Days	27	a
Employee	4	12

$$\frac{27}{a} = \frac{12}{4}$$

a = 9 days

Multiple Choice Questions (M.C.Q.)

1.

Books	20	b
Cost	280	700

$$\frac{20}{280} = \frac{b}{700}$$

$$\frac{20 \times 700}{280} = b$$

b = 50 books (c)

2. 24 boards = 60 books

40 boards = a

$$a = \frac{60 \times 40}{24} = 100 \text{ books}$$

100 books are required. (a)

3. 20 oranges = ₹70

x oranges = ₹ 175

$$x = \frac{20 \times 175}{70} = 50 \text{ oranges (a)}$$

4.

Cost	144	x
Men	12	24

$$\frac{144}{12} = \frac{x}{24}$$

144 × 2 = x

x = ₹288 (d)

5.

Student	500	600
Day	30	a

$$\frac{500}{600} = \frac{a}{30} \qquad \Rightarrow \frac{500 \times 30}{600} = a$$

a = 25 days (a)

6.

Oxen	6	9
Days	30	a

$$\frac{6}{9} = \frac{a}{30}$$

$$\frac{6 \times 30}{9} = a$$

a = 20 (b)

7. (c) The speed of a bus and time taken to cover a distance

11 Time and Work

1. Time taken by 1st boy = 5 days
   Time taken by 2nd boy = 4 days
   Work done by 1st in 1 day $= \frac{1}{5}$
   Work done by 2nd in 1 day $= \frac{1}{4}$
   Total $\Rightarrow \frac{1}{5} + \frac{1}{4} = \frac{9}{20}$
   So, both can do the work in $= \frac{20}{9}$ days.
2. Time taken by A and B to do the work = 10 days
   Time taken by A alone to do the work = 15 days
   Work done by A and B in 1 day $= \frac{1}{10}$
   Work done by A in 1 day $= \frac{1}{15}$
   Work done by B in 1 day $= \frac{1}{10} - \frac{1}{15} = \frac{3-2}{30} = \frac{1}{30}$
   So, B can do work in 30 days.
3. 'x', 'y' and 'z' work together can do piece of works = 8 hour
   x alone can done work in 1 day $= \frac{1}{20}$

y can done work in 1 day $= \frac{1}{24}$

So, z can done work in 1 day $= \frac{1}{8} - \frac{1}{20} - \frac{1}{24} = \frac{1}{30}$

So, z can do work alone in 30 days

4. Together they take 8 days

Kriti can do work in 1 day $= \frac{1}{24}$

Naman can do work in 1 day $= \frac{1}{20}$

Riya can done work in 1 day $= \frac{1}{8} - \frac{1}{20} - \frac{1}{24} = \frac{1}{30}$

Riya can do work in 30 days.

5. Amit can do work in 1 day $= \frac{1}{12}$

Megha can do the alone in 1 day $= \frac{1}{20}$

They together do $= \frac{1}{12} + \frac{1}{20} = \frac{5+3}{60} = \frac{8}{60} = \frac{2}{15}$

As for 3 day they work together $= 3 \times \frac{2}{15} = \frac{2}{5}$

Remaining work $= 1 - \frac{2}{5} = \frac{3}{5}$

The remaining work is completed in 20 days

$\therefore$ Megha doing $\frac{1}{20}$ work $= \left(20 \times \frac{3}{5}\right) = 12$ days

remaining work in 12 days.

6. L and M = 10 days

M and N = 12 days

L and N = 15 days

(L and M)'s 1 day work $= \frac{1}{10}$

(M and N)'s 1 day work $= \frac{1}{12}$

(L and N)'s 1 day work $= \frac{1}{15}$

Adding, we get 2(L + M + N)'s 1 day work

$$\Rightarrow \frac{1}{10}+\frac{1}{12}+\frac{1}{15}=\frac{6+4+5}{60}=\frac{15}{60}$$

L + M + N's in 1 day $= \frac{15}{60}\times\frac{1}{2}=\frac{15}{120}=\frac{1}{8}$

Together L, M, N can finish the work in 8 days.

As 1 day work =

(L + M + N)'s 1 day work – (L + N)'s 1 day work

$$=\frac{1}{8}-\frac{1}{15}=\frac{7}{120}$$

M alone take $17\frac{1}{7}$ days

L alone 1 day $= \frac{1}{8}-\frac{1}{12}=\frac{1}{24}$ or 24 days

N 1 day works $= \frac{1}{8}-\frac{1}{10}=\frac{1}{40}$ or 40 days to complete.

7. 1 man work in 6 hours.
   2nd man work in 4 hours.

   For 1 hour it work $= \frac{1}{6}$

   For 2 hour $= \frac{1}{3}$

   And both also $= \frac{1}{6}+\frac{1}{4}=\frac{2+3}{12}=\frac{5}{12}$

   Remaining work $= 1-\frac{1}{3}=\frac{2}{3}$

   So $\frac{2}{3}$ work $= \frac{3}{2}\times\frac{5}{12}=\frac{5}{8}$

   So remaining work will be completed in $= \frac{8}{5}=1\frac{3}{5}$ days

8. Three men P, Q, R
   P work = 9 hour

Q work = 18 hour
R work = 12 hour

Their together work for 1 hour $= \frac{1}{9}+\frac{1}{18}+\frac{1}{12}=\frac{1}{4}$

It took 4 hours.

9. A, B and C together do work in 10 days
A alone can do it in 40 days
B alone can do it = 30 days

$$\text{For C} = \frac{1}{10}-\frac{1}{40}-\frac{1}{30}=\frac{12-3-4}{120}$$
$$=\frac{5}{120}=24$$

C can do it in 24 days.

10. By one tap in 4 hour
Another tap in 3 hour

Both opened then $= \frac{1}{4}+\frac{1}{3}=\frac{7}{12}$

Or, by $\frac{12}{7}$ hour or $1\frac{5}{7}$ hour

11. Work done by A tank in 1 hour $= \frac{1}{6}$

Work done by B tank in 1 hour $= \frac{1}{8}$

Both can do $= \frac{1}{6}+\frac{1}{8}=\frac{14}{48}=\frac{7}{24}$

But in 2 hour A and B can do $= \frac{2\times 7}{22}=\frac{7}{12}$

Remaining $= 1-\frac{7}{12}=\frac{5}{12}$

By B $= 8\times\frac{5}{12}=\frac{10}{3}=3\frac{1}{3}$ hours

12. Tank is filled at the rate of = 6 hour

In 1 hour $= \frac{1}{6}$

Compled by = 7 hour

In 1 hour $= \frac{1}{7}$

Then overall work in 1 hour will be $= \frac{1}{6} - \frac{1}{7} = \frac{1}{42}$ hours

So in 42 hour it will complete.

13. Time taken by tap 1st to fill = 12 min

Time taken by 2nd to fill = 15 min

Time taken by 3rd to empty = 10 min

Thus in 1minute the overall work will be $= \left(\frac{1}{12} + \frac{1}{15} - \frac{1}{10}\right)$

$$= \left(\frac{5+4-6}{60}\right) = \frac{1}{20}$$

So 20 minutes will required.

14. Work done by tap A in 1 hour $= \frac{1}{10}$

By tap B in 1 hour $= \frac{1}{15}$

Both in 1 hour $= \frac{1}{10} + \frac{1}{15} = \frac{3+2}{30} = \frac{1}{6}$

In 4 hour $= \frac{4}{6} = \frac{2}{3}$

Remaining $= 1 - \frac{2}{3} = \frac{1}{3}$

By A $= 10 \times \frac{1}{3} = \frac{10}{3}$

$3\frac{1}{3}$ hour it take

15. Work done by tap in 1 hour $= \frac{1}{4}$

Work done by pipe in 1 hour $= \frac{1}{6}$

Work done by both in 1 hour $= \frac{1}{4} - \frac{1}{6} = \frac{3-2}{12} = \frac{1}{12}$

So, time taken to fill the tank by these two = 12 hours

Check Your Mental Math I.Q.

1. Less.
2. Negative
3. In 1 day work done $= \frac{1}{20}$

   In 4 days $= \frac{4}{20} = \frac{1}{5}$
4. Together $= \frac{1}{15} + \frac{1}{10} = \frac{2+3}{30} = \frac{5}{30} = \frac{1}{6}$

   So both can work in 6 hour.
5. A can do $= \frac{1}{8} - \frac{1}{10} = \frac{5-4}{40} = \frac{1}{40}$ in 40 days
6. $\frac{1}{12} + \frac{1}{16} - \frac{1}{8} = \frac{4+3-6}{48} = \frac{1}{48}$

   Or in 48 hour it can empty the whole tank.

Multiple Choice Questions (M.C.Q.)

1. Complete work in 12 days

   1 day $= \frac{1}{12}$

   In 9 day $= \frac{9}{12} = \frac{3}{4}$ (b)
2. Ruchi alone can do $= \frac{1}{20} - \frac{1}{30} = \frac{3-2}{60} = \frac{1}{60}$ 60 days (b)
3. All three $= \frac{2}{3} + \frac{1}{2} + \frac{1}{3} = \frac{4+3+2}{6} = \frac{9}{6}$

   $= \frac{3}{2}$ hour $\Rightarrow \frac{2}{3}$ hours (a)
4. 60 men work in 8 hours, build shed in 30 days

   If there were 120 men work for 6 hour then same work is to be done in

   $= \frac{60 \times 8 \times 30}{120 \times 6} = 20$ days (b)
5. $\frac{1}{12} - \frac{1}{18} = \frac{3-2}{36} = \frac{1}{36} = 36$ hours (b)

12 Geometry

Exercise 12

1. (a) 4, PQ, QR; QR, RS; RS, SP; SP, PQ
   (b) 2: PQ, SR; PS, QR
   (c) 2; $\angle P, \angle Q$; $\angle Q, \angle R$; $\angle R, \angle S$; $\angle S, \angle P$
   (d) 2; $\angle P, \angle R$; $\angle Q, \angle S$
2. Let the two equal angles be x

   So, $60 + 100 + x + x = 360°$

   $2x = 200$

   $x = 100°$

   Angle are 100, 100
3. Let the ratio be x

   Then $2x+3x+5x+8x = 360$

   $18x = 360$

   $x = 20$

   So, angles are 40°, 60°, 100°, 160°
4. (i) $120 + 50 + 130 + x = 360°$

   $x = 360° - 300$

   $x = 60°$

   (ii) $72 + 25 + 30 + x = 360°$

   $x = 233°$

   (iii) $180° - 120° = x$

   $60° = x$

   $y = 180 - 80 = 100°$

   $z = 180 - 60 = 120°$

   $80°+120°+60°+(180-w) = 360°$

   $w = 80°$

   So, $x = 60°$, $y = 120°$, $z = 120°$, $w = 80°$
5. Let the angle be = x

   $4x = 360°$

   $x = 90°$

6. $130+65+65+x = 360°$

$x = 100°$

7. $\angle A+\angle B+100+50=360°$

$\angle A+\angle B = 360°-150=210$

$$\frac{\angle A+\angle B}{2}=105°=\frac{210}{2}=105°$$

So, $\angle APB = 180°-\left(\frac{\angle A+\angle B}{2}\right)$

$\angle APB = 180°-105°=75°$

Check Your Mental Math I.Q.

1. Let the angle be x

$4x = 360°$

$x = 90°$

2. Let the ratio be x the

then $3x+5x+7x+9x=24x=360°$

$x = 15°$

So the angles are $=45°, 75°, 105°, 135°$

3. Let the ratio be x

$4x+6x+3x+100=360°$

$13x = 260°$

$x = 20°$

$4\times20=80$; $6\times20=120$ $3\times20=60$

4. Let the equal angle be x = then

$3x+120° = 360°$

$3x = 240°$

$x = 80°$

5. $2x°+10, 3(x+2)°, (7x-10)°, 6(x+5)°$

$2x+10+3x+6+7x-10+6x+30=360°$

$18x = 360-36=324$

$x = 324\div18=18$

$2\times18+10=46°$ $3(18+2)=60°$

$(7\times18-10)=116°$ $6(x+5)=6(18+5)$

$=6\times23=138°$

Multiple Choice Questions (M.C.Q.)

1. (b) $360°$ 2. (b) $60°$ 3. (b)
4. (a) $2x + 3x + 4x + 6x = 360°$, then $x = 24$. So angles are 48, 72, 96, 144
5. $5x + x = 360°$
   $x = 60°$ (b)

13 Special Types of Quadrilaterals

Exercise 13A

1. A quadrilateral which has exactly one part of parallel sides, in an isoceles trapezium non parallel sides are equal.
2. $$\begin{aligned} 60 + \angle B &= 180° \\ \angle B &= 120° \\ \angle A &= \angle C \\ \angle C &= 60° \\ \angle B &= \angle D \\ \angle D &= 120° \end{aligned}$$
3. If two angles are equal then
$$\begin{aligned} x + x &= 180° \\ 2x &= 180° \\ x &= 90° \end{aligned}$$
4. $$\begin{aligned} \angle y &= 180 - 100 \\ &= 80° \\ \angle x &= \angle 100° \\ \angle y &= \angle z \qquad \textit{i.e.}\ \angle z = 80° \end{aligned}$$
5. $$\begin{aligned} \angle c = \angle A &= 75° \\ \angle CDB &= 180° - 75° - 60° = 45° \\ \angle ADB &= \angle CBD = 60° \end{aligned}$$

6. $$25+25+x+x = 150°$$
$$2x = 100$$
$$x = 50°$$
So, the sides are 25, 50, 25, 50

7. Let the ratio of x
$$x+3x = 180°$$
$$4x = 180°$$
$$x = 45°$$
or $$3x = 45 \times 3 = 135°$$
Angles are = 45°, 135°, 45°, 135°

8. $x+x+30+x+x+30 = 180$
$$4x = 180-60$$
$$4x = 120$$
$$x = 30; \text{So sides are } 30, 60, 30, 60$$

9. $$\angle 40 + \angle C = 70°$$
$$\angle C = 30°$$
$$\angle a + 70° = 180°$$
$$\angle a = 110°$$
In WYX
$$\angle C + \angle B =$$
$$\angle B = 70-30°$$
$$\angle B = 40°$$

Exercise 13B

1. $a = 5$ cm $\quad b = 12$ cm
$c^2 = 5^2 + 12^2 = 169; c = 13$ cm

2. $$BO = \frac{1}{2} BD$$
i.e. $$BO = 8$$
$$AO = \sqrt{(10)^2 - (8)^2}$$
$$AO = \sqrt{100-64}$$
$$AO = \sqrt{36}$$
$$AO = 6$$

$$AO = \frac{1}{2} \times AC$$

$$6 \times 2 = AC$$

$$AC = 12 \text{ cm}$$

3. $\angle DBC = 90 - \angle BAC$

$= 90 - 28 = 62°$

4. $\angle ABD = \frac{1}{2} \angle AC$

$\angle ABD = \frac{1}{2} \times 126 = 63$

$\angle ACD = 90 - 63 = 27$

5. $\angle DCA = \frac{90}{2} = 45°$

6. (i) $\angle DAO = \angle CBO$

$\angle DAO = 90 - 21° = 69°$

$69° = x°$

(ii) $\angle b = \frac{180 - 102}{2} = \frac{78}{2} = 39°$

$\angle a + \angle b = 90°$

$\angle a = 90 - b = 90° - 39° = 51°$

(iii) $180 - 140 = 2x$

$20° = x$

7. $\angle A = \angle C = 60°$

$\angle B = \angle D = 120°$

8. $\angle ODA = \frac{180 - 60}{2} = \frac{120}{2} = 60°$

Check Your Mental Math I.Q.

1. Let the ratio be x

$5x + 3x + 3x + 5x = 360°$

$16x = 360°$

$x = 22.5°$

$3x = 3 \times 22.5 = 67.5$

$5x = 22.5 \times 5 = 112.5$

2. $3x + 5x + 3x + 5x = 48$

$16x = 48 \Rightarrow x = 3$

$3 \times 3 = 9$; $5 \times 3 = 15$

3. $4x = 120$

   $x = 30$ cm

4. $15 + 15 + 2x = 80$

   $2x = 50$

   $x = 25$

5. PQS $= 50°$

   SQP $= 90 - 50° = 40°$

6. (a) False (b) True (c) False (d) False
   (e) False (f) False (g) True (h) False

Multiple Choice Questions (M.C.Q.)

1. (a) rectangle
2. (b) Square
3. (a) Trapezium
4. (c) each other
5. (a) Square
6. (a) parallelogram
7. (a) 60°, 128°, 120°
8. (c) $180 - 70° = 110°$

15 Solid Shapes

2. (a) Tetrahedron (b) Cube, cuboid
   (c) Square pyramid and rectangular pyramid
   (d) Regular octahedron (e) Cylinder
3. No
5. 8; 6; 30; 4
6. (a) Square (b) Circle and rectangle
   (c) Triangle and rectangle (d) Triangle
7. (a) Triangular pyramid (b) Hexagonal pyramid
   (c) Hexagonal prism (d) Triangular prism
   (e) Triangular prism (f) Square pyramid
   (g) Square pyramid (h) Cube

Check Your Mental Math I.Q.

1. (a) False (b) True (c) True (d) False
   (e) False (f) False (g) True
2. 10
3. 2

Multiple Choice Questions (M.C.Q.)

1. (c)
2. (a)
3. (b)
4. (a)

16 Area of a Trapezium and a Polygan

1. Area of Trapezium =

$$= \frac{1}{2} \times (15+8) \times 10$$

$$= 23 \times 5 = 115\text{cm}^2$$

2. $\frac{1}{2} \times (40+22) \times 12$

Area of garden $= \frac{1}{2} \times (62) \times 12$

$$= 62 \times 6 = 372\text{m}^2$$

3. $240 = \frac{1}{2} \times (25+x) \times 10$

$$\frac{2 \times 240}{10} = 25 + \text{x}$$

$$48 - 25 = \text{x}$$

$$\text{x} = 23\text{cm}$$

4. $500 = \frac{1}{2} \times (50) \times h$

$$\frac{1000}{50} = \text{h}$$

$$\text{h} = 20\text{ cm}$$

5. $210 = \frac{1}{2}(\text{x}+2\text{x}) \times 14$

$$\frac{210 \times 2}{14} = 3\text{x}$$

$$\text{x} = 10 \text{ and } 3\text{x} = 30$$

6. The length of the parallel sides of a trapezium ratio be x as 5x, 6x

$$330 = \frac{1}{2} \times (5x+6x) \times 12$$

$$\frac{660}{12\times 11} = x$$

$$x = 5$$

So, sides are $5\times 5 = 25$

$6\times 5 = 30$

7. $$192 = \frac{1}{2}\times(8+x)\times 16$$

$$\frac{192\times 2}{16} = x+8$$

$$x = 24-8$$

$$x = 16$$

So, Other side is 16

8. Let one is x then other is $7+x$

$$138 = \frac{1}{2}\times(x+7+x)\times 12$$

$$\frac{138\times 2}{12} = 7+2x$$

$$16 = 2x$$

$$8 = x$$

So one is 8 and the other is $8+7=15$

10. $$180 = \frac{1}{2}\times(x+x+6)\times 9$$

$$\frac{180\times 2}{9} = 2x+6$$

$$40-6 = 2x$$

$$\frac{34}{2} = 2x$$

$$x = 17$$

$17, 17+6=23$

11. $$\text{Area} = \frac{1}{2}\times(10+25)\times 15$$

$$= \frac{35\times 1}{2}\times 15$$

$$\text{Cost} = \frac{35\times 1}{2}\times 15\times 4$$

$= ₹\,1050$

12. Perimeter of Trepezium = 52 cm
Sum of unparallel sides = 10 + 10 = 20
Sum of parallel sides = 52 – 20 = 32 cm

So, Area of Trepezium = $\frac{1}{2} \times (32) \times 8$

$= 128\ \text{cm}^2$

Exercise 16B

1. Area of polygon A $= \frac{1}{2} \times$ perimeter $\times$ in radius

Perimeter $= na$

radius $= r$

$$A = \frac{1}{2} nar$$

$A = \frac{1}{2} \times 5 \times 10 \times 9$ {as pentagon so n = 5}

$= 225\text{cm}^2$

2. $$A = \frac{1}{2} \times na \times \sqrt{R^2 - \frac{a^2}{4}}$$

$$= \frac{1}{2} \times 7 \times 8 \times \sqrt{(6)^2 - \frac{(8)^2}{4}}$$

$$= 28\sqrt{36 - 16}$$

$$= 28 \times \sqrt{20}$$

$$= 125.22\ \text{cm}^2$$

3. $$A = \frac{1}{2} \times na \times \sqrt{R^2 - \frac{a^2}{4}}$$

$$= \frac{1}{2} \times 9 \times 6 \times \sqrt{(4)^2 - \frac{(6)^2}{4}}$$

$$= 9 \times 3 \times \sqrt{16 - 9} = 27\sqrt{7} = 71.435 cm^2$$

4. Area of regular hexagon

$$= \frac{3\sqrt{3}}{2} \text{ (side)}^2 \text{ sq unit}$$

$$= \frac{3\sqrt{3}}{2} \times (7)^2 = \frac{3\sqrt{3}}{2} \times 49 = 127.302 cm^2$$

(b) $\frac{3\sqrt{3}}{2} \times (9)^2 = \frac{3\sqrt{3}}{2} \times 81 = 210.438 cm^2$

5. Area of regular octagon $= 2a^2\left(1+\sqrt{2}\right)$

$$= 2 \times (8)^2\left(1+\sqrt{2}\right)$$

$$= 2 \times 64\left(1+\sqrt{2}\right) = 308.992$$

(b) $= 2 \times (5)^2\left(1+\sqrt{2}\right)$

$$= 2 \times 25\left(1+\sqrt{2}\right)$$

$$= 120.7 \text{cm}^2$$

6. (a) Area of ΔAFB $= \left(\frac{1}{2} \times B \times h\right) = \frac{1}{2} \times 20 \times 10$

$$= 10 \times 10 = 100 \text{ cm}^2$$

Area of square $= (s)^2 = (20)^2 = 400 \text{ cm}^2$

Area of ΔECD $=$

$$OD^2 = (20)^2 - (10)^2$$

$$OD^2 = (400 - 100)$$

$$OD = 10\sqrt{3}$$

$$= \left(\frac{1}{2} \times 20 \times 10\sqrt{3}\right) = 100\sqrt{3} cm^2$$

Total Area $= 100\sqrt{3} + 100 + 400$

$$= 673.20 \text{m}^2$$

(b) PD $= \sqrt{(8)^2 - (4)^2}$ $\qquad \sqrt{(12)^2 - (4)^2}$

$= \sqrt{64 - 16}$ $\qquad \sqrt{144 - 16}$

$= \sqrt{48}$ $\qquad \sqrt{128}$

$$= 4\sqrt{3}$$

$$\text{Area of FDA} = \left(\frac{1}{2}\times 4\times 4\sqrt{3}\right)+\left(\frac{1}{2}\times 4\times\sqrt{128}\right)$$

$$= 8\sqrt{3}+2\sqrt{128}$$

$$\text{DQ} = \sqrt{(12)^2-(5)^2} \qquad \sqrt{(12)^2-(5)^2}$$

$$= \sqrt{144-25}$$

$$= \sqrt{119}$$

$$\text{Area } \nabla\text{AB} = \left(\frac{1}{2}\times 5\times\sqrt{119}\right)\times 2$$

$$= 5\sqrt{119}$$

$$\text{DR} = \sqrt{(12)^2-(5)^2} = \sqrt{119}$$

$$= \sqrt{(10)^2-(5)^2} = \sqrt{100-25} = \sqrt{75}$$

$$\text{So area of DBC} = \left(\frac{1}{2}\times\sqrt{119}\times 5\right)+\left(\frac{1}{2}\times\sqrt{75}\times 5\right)$$

$$= \frac{5\sqrt{119}}{2}+\frac{5\sqrt{75}}{2}$$

$$\text{Total area} = 8\sqrt{3}+2\sqrt{128}+5\sqrt{119}+\frac{5\sqrt{119}}{2}+\frac{5\sqrt{75}}{2}$$

$$= 134.29\text{cm}^2$$

7. $MO = \sqrt[2]{\left(13^2-5^2\right)} = \sqrt[2]{(169-25)} = 2\times 12 = 24m$

$$\text{Area of figure} = 2\left(\frac{1}{2}\times 24\times 5\right)+13\times 24$$

$$= 120+312 = 432 \text{ sqm}$$

8. Area of pentagon ABCDE

Area of EHD + Area of trapezium HDCF + Area of Δ CFB + area of Δ AEB.

As

$$\frac{1}{2}\times(EH\times HD)+(DH+CF)\times HF+\frac{1}{2}\times CP\times FB+\frac{1}{2}(EB\times AG)$$

$EH = BE - BH = 120 - 80 = 40$

$HF = BH - BF = 80 - 20 = 60$

$$\frac{1}{2} \times (40 \times 25) + \frac{1}{2}(25 \times 30) \times 60 + \frac{1}{2} \times 30 \times 20 + \frac{1}{2}(120 \times 70)$$

$= 6650 m^2$

9. Area of ABCDEFG =
   Area of AGL + Area of Trapezium GLFN + Area of GNE + Area of Trapezium ABCU + Area of lrapism UCDN + Area of ODE

   $LN = AO - AL = 110 - 30 = 80, NE = AE - AO = 25$

$$\frac{1}{2} \times AL \times LG + \frac{1}{2}(GL + FN)LN + \frac{1}{2} \times (FN) \times (NE)$$

$$+ \frac{1}{2} \times AU \times (AB + UC) + \frac{1}{2} \times MO \times (MC + OD) + \frac{1}{2} \times (OE) \times (a)$$

$$\frac{1}{2} \times 30 \times 40 + \frac{1}{2}(40 + 42) \times 80 + \frac{1}{2} \times 42 \times 25 +$$

$$\frac{1}{2} \times 90 \times (36 + 54) + \frac{1}{2} \times (54 + 46) \times 20 + \frac{1}{2} \times 25 \times 40$$

$= 93730 m^2$

Check Your Mental Math I.Q.

1. Area $= \frac{1}{2}\ (9 + 19) \times 11 = 154\, m^2$

2. $$340 = \frac{1}{2} \times 8 \times (8 + x)$$
   $$\frac{340 \times 2}{8} = 8 + x$$
   $$85 = 8 + x$$
   $$x = 77$$

3. $$22 = \frac{1}{2} \times 11 \times h$$
   $$\frac{22 \times 2}{11} = h$$
   $$h = 4\, cm$$

4. Let the ratio be x

So, the sides are = 3x, 5x

$$240 = \frac{1}{2} \times (3x + 5x) \times 12$$

$$\frac{240 \times 2}{12 \times 8} = x$$

$$x = 5$$

$3 \times 5 = 15$ cm; $5 \times 5 = 25$ cm

5. Area of square = $(s)^2$

Area of rectangle = $(l \times b)$

Perimeter of square = $4 \times s$

Perimeter of rectangle = $2(l + b)$

$$\frac{(S)^2}{15 \times 9} = \frac{3}{5}$$

$$(s)^2 = 27 \times 3$$

$$S = \sqrt{81}$$

$$S = 9$$

Perimeter of square = $4 \times 9 = 36$ cm

Multiple Choice Question (M.C.Q.)

1. $A = \frac{1}{2} \times 10 \times 7 = 35 cm^2$ (c)

2. $36 = \frac{1}{2} \times 18 \times D_1$

$4 = D_1$

Perimeter = $4 \times 4 = 16$ cm (a)

3. $$475 = \frac{1}{2} \times (2x + 3x) \times 10$$

$$\frac{475 \times 2}{5 \times 10} = x$$

$$x = 19$$

Larger side = $3 \times 19 = 57$ cm (b)

4. $A = \frac{1}{2} \times (80 + 60) \times 20 = 1400$ cm^2 (a)

17 Volume and Surface Area of Solids

1. C.S.A. of cylinder $= 2 \times \pi \times r \times h$

$= 2 \times \frac{22}{7} \times 7 \times 15$

$= 660 cm^2$

T.S.A. of cylinder $= 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 7(15 + 7)$

$= 968\ cm^2$

2. C.S.A. of cylinder $= 2 \times \pi r h$

$88 = 2 \times \frac{22}{7} \times r \times 14$

$\frac{88 \times 7}{22 \times 14} = 2r$

$D = 2\ cm \quad \{\therefore D = 2r\}$

3. Volume $= \pi r^2 h$

$= \frac{22}{7} \times 3.5 \times 3.5 \times 8$

$= 308\ cm^3$

4. Volume $= \pi r^2 h$

$3080 = \frac{22}{7} \times r^2 \times 20$

$\frac{3080 \times 7}{20 \times 22} = r^2$

$r^2 = 49; \quad \Rightarrow \quad r = 7\ cm$

5. Volume $= \pi r^2 h$

Given $D = 14m$; So radius $= \frac{14}{2} = 7$ m

$2156 = \frac{22}{7} \times (7)^2 \times h$

$$\frac{2156}{7\times 22} = h$$

$$h = 14m$$

6. C.S. of roller $= 2\pi rh$

$$= 2\times\frac{22}{7}\times 60\times 84$$

$$= 31680\ cm^2$$

1000 revolutions then area

$$= 31680\times 1000 = 31680000\ cm^2$$

$$= 3168\ m^2$$

7. Orginal cylinder volume $= \pi r^2 h$

Reduced cylinder volume $= \pi\times\left(\frac{r}{2}\right)^2 h$

$$\text{Ratio} = \frac{\pi\times r^2\times h}{4(\pi\times r^2\times h)} \qquad i.e.\ \frac{reduce}{original}$$

$$= 1:4,\ \text{Ans}$$

8. 1 pilar C.S.A. $= 2\pi\times r\times h$

$$= 2\times\frac{22}{7}\times\frac{0.48}{2}\times 7$$

$$= 22\times 0.48\ m^2$$

1 pillar cost of painting $= 22\times 0.48\times 5$

12 pillar cost of painting $= 22\times 0.48\times 5\times 12$

i.e. $= ₹\ 633.60$

9. $h = 22.5m;\ d = 7m$ or $r = \frac{7}{2}$

Inner C.S.A. $= 2\pi\times r\times h$

Area dug $=$ Volume $= \pi r^2 h$

$$= \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times 22.5$$

$$= 866.25\ cm^3$$

C.S.A. $= 2\times\frac{22}{7}\times\frac{7}{2}\times 22.5$

C.S.A. of $= 22 \times 22.5\ m^2 = 495\ m^2$

Cost of plastering $= 3 \times 495 =$ ₹1485

10. Volume of 1 coin $= \pi r^2 h$

$$= \frac{22}{7} \times \frac{1.5}{2} \times \frac{1.5}{2} \times 0.2 cm^3$$

Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times \frac{4.5}{2} \times \frac{4.5}{2} \times 10$$

$$\text{No. of Coins} = \frac{\text{Volume of cylinder}}{\text{Volume of cone}}$$

$$= \frac{22/7 \times 4.5 \times 4.5 \times 10}{22/7 \times 1.5 \times 1.5 \times 0.2}$$

$= 450$ coins are required

11. Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times \frac{20}{2} \times \frac{20}{2} \times 9$$

$= 2828.60 cm^3$

Volume of cube $= (8cm)^2$

$= 512\ cm^3$

$$\text{heigh} = \frac{\text{Volume of cylender}}{\text{Volume of cube}}$$

$$= \frac{2828.6}{512}$$

$= 5.52$ cm

12. C.S.A. of pipe $= 2 \times \frac{22}{7} \times \frac{70}{2} \times 21 \times \frac{1}{100}$

$= 46.2\ cm^2$

Cost $= 46.2 \times 10 =$ ₹462

13. as height is same

Volume of hollow cylinder = volume of solid cylinder

$\pi(R^2 - r^2) \times h = \pi \times R_1 \times h$

$$\frac{\pi\left(R^2 - r^2\right)\times h}{\pi \times h} = R_1^2$$

$$R^2 = (20)^2 - (15)^2$$

$$R_1^2 = 175$$

$$R_1 = 13.3 \text{ cm approx.}$$

So radius of solid cylinder is 13.3 cm

14. (i) Volume of roof $= l \times b \times h$

$$= 70 \times 44 \times \frac{1}{10}$$

$$= 308 \text{ cm}^3$$

(ii) Volume of tank $= \pi r^2 h$

$$= \frac{22}{7} \times 14 \times 14 \times h$$

$$= 61.6 \text{ h m}^3$$

Rise in volume $= \dfrac{308}{61.6} = 5 \text{ m}$

15. $2\pi r = 22$ or $r = \dfrac{22 \times 7}{22 \times 2}$

$r = \dfrac{7}{2}$

Now volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12$$

$$= 462 \text{ cm}^3$$

Check Your Mental Math I.Q.

1. Volume of cuboid $= 6 \times 3 \times 4 = 72 \text{ cm}^3$
2. $24 = 4 \times 3 \times h$

$$\frac{24}{4 \times 3} = h$$

$$h = 2 \text{ m}$$

3. Volume of 5 small cubes = volume of cube

Let edge be x then

$$x^3 = \frac{135}{5}$$

$$x^3 = 27$$

$$x = 3$$

4. No of cubical block $= \dfrac{\text{Volume of block}}{\text{Volume of small block}}$

$$= \frac{(6)^3}{(2)^3}$$

$$= \frac{6 \times 6 \times 6}{2 \times 2 \times 2}$$

27 cubical block.

5. Volume of cylinder = volume of 2nd cylinder

$$\pi \times R^2 \times H = \pi \times r^2 \times h$$

$$\frac{H}{h} = \frac{\pi \times (r)^2}{\pi \times (R)^2}$$

$$\frac{H}{h} = \left(\frac{3}{5}\right)^2$$

$$\frac{H}{h} = \frac{9}{25}$$

So, $\dfrac{h}{H} = \dfrac{25}{9}$ or $25 : 9$

Multiple Choice Questions (M.C.Q.)

1. (c)

2. $$\text{S.A.} = 486$$
$$6a^2 = 486$$
$$a^2 = 81$$
$$a = 9 \text{ (b)}$$

3. $$\text{S.A.} = 6a^2$$
$$= 6 \times (5)^2$$
$$= 150 \text{ (b)}$$

4. No. of cubes $= \dfrac{\text{Volume of block}}{\text{Volume of small block}}$

$$= \frac{(8)^3}{(2)^3}$$

$$= (4)^3 = 64 \quad \text{(a)}$$

5. Length $= 4 \times 3 = 12\text{m}$

Breadth $= 4\text{ cm}$

S.A. $= 4(12 \times 4) + 2(4 \times 4)$

$192 + 32 = 224\text{ cm}^2$ (b)

6. as of $= \dfrac{\pi r^2 \times h}{\pi R_2^2 \times H}$

$$= \left(\frac{r}{R}\right)^2 \left(\frac{h}{H}\right)$$

$$= \left(\frac{2}{3}\right)^2 \left(\frac{5}{3}\right)$$

$$= \frac{4}{9} \times \frac{5}{3} = \frac{20}{27} \quad \text{(c)}$$

18 Data Handling

Check Your Mental Math I.Q.

1. data
2. raw data
3. grouped
4. range
5. class interval

Multiple Choice Questions (M.C.Q.)

1. (a) 15
2. (b) 30
3. (c) 5
4. (d) 32.5
5. (c) 6

19 Graphical Representation of Data

Exercise 19A

4. (a) It depicts the height of 20 students of a class.
   (b) 155 cm (c) 11 students
   (d) 3 students
5. (a) 12 (b) 13 matches
   (c) 8 matches (d) 4 matches
6. (a) 30 - 35 (b) 6
   (c) 5
   (d) 20 - 25; 25 - 30; 30 - 35; 35 - 40; 40 - 45; 45 - 50

Exercise 19B

3. (a) In steel $= \dfrac{540000 \times 45}{360} =$ ₹ 67500

   Cement $= \dfrac{540000 \times 75}{360} =$ ₹ 112500

   (b) Timber $= \dfrac{540000 \times 50}{360} = 135000$

   Bricks $= \dfrac{540000 \times 50}{360} = 75000$

   Difference $= 135000 - 75000 = 60000$

4. (a) Wheat $= \dfrac{81000 \times 120}{360} = 27000$ tonnes

   (b) Sugar $= \dfrac{81000 \times 100}{360} = 22500$ tonnes

   (c) Rice $= \dfrac{81000 \times 60}{360} = 13500$ tonnes

   (d) Grain $= \dfrac{81000 \times 50}{360} = 11250$ tonnes

   (e) Maiza $= \dfrac{81000 \times 30}{360} = 6750$ tonnes

Check Your Menatl Math I.Q.

1. Histogram
2. One dimensional
3. pie chart
4. 360°

Multiple Choice Questions

1. (b) $\frac{24 \times 60}{360} = 4$
2. (b) $\frac{60 \times 60}{360} = 10$
3. (b) $\frac{36 \times 60}{360} = 6$
4. (b) $\frac{360 \times 30}{360} = \frac{210 \times 60}{360} = \frac{30}{210}$

$\frac{1}{7} = 1:7$

20 Probablity

1. Total number of event = 6
   (a) an even number face favourable = 2, 4, 6 i.e. 3

   Probality = $\frac{3}{6} = \frac{1}{2}$

   (b) a multiple of 3
   favourable = 3 , 6 i.e. 2

   Probablity = $\frac{2}{6} = \frac{1}{3}$

   (c) an odd number favourable = 1, 3, 5, i.e. 3

   Probability = $\frac{3}{6} = \frac{1}{2}$

   (d) a number between 3 and 6
   favourable = 4, 5 i.e. 2

   Probablity = $\frac{2}{6} = \frac{1}{3}$
2. Total number of events = $2^3 = 8$

(a) All head

Favourable = 1 Probablity = $\frac{1}{8}$

(b) two head

favourable = 3 Probablity = $\frac{3}{8}$

(c) one head

Favourable = 3 Probablity = $\frac{3}{8}$

(d) at least two heads = 4

Favourable = 4 Probablity = $\frac{4}{8} = \frac{1}{2}$

HHH
HTH
HTT
THH
TTH
THT
TTT

3. Total number of possiblity = 25
   Favourable event = 1, 3, 5, 7, 11, 13, 17, 19, 23 = 9
   Probablity = $\frac{9}{25}$
4. Total number of events = 52
   (a) an ace

   Favourable event = 4 Probablity = $\frac{4}{52} = \frac{1}{13}$

   (b) Black

   Favourable event = 26 Probablity = $\frac{26}{52} = \frac{1}{2}$

   (c) either black card or king

   Favourable event = 28 = 26+2 Probablity = $\frac{7}{13}$

   (d) a face card

   Favourable event = $3 \times 4 = 12$ Probablity = $\frac{12}{52} = \frac{3}{13}$

   (e) a red face card = 6

   Favourable event = 6 Probablity = $\frac{6}{52} = \frac{3}{26}$

5. Total number of events = 16

   Favourable event = 7 Probablity = $\frac{7}{16}$

6. Total number of ball = 20
   Favourable event for getting white = 5

   Probablity of white = $\frac{5}{20}$

   Probablity of getting non white = $\frac{20-5}{20} = \frac{15}{20}$

7. Total number of events = $6 \times 6 = 36$
   (a) outcome for odd
   Favourable events = (1, 6), (3, 6), (5, 6), (2, 5), (4, 5), (6, 5), (1, 4), (3, 4), (5, 4), (2, 3), (4, 3), (6, 3), (1, 2), (3, 2), (5, 2), (2, 1), (4, 1), (6, 1)

   (b) $\frac{18}{36}$ *or* $\frac{1}{2}$

   (c) (2, 6), (3, 5), (4, 4), (5, 3), (6, 2)

   (d) Probablity of total 8

   Favourable event = 5 Probability = $\frac{5}{36}$

8. Total number of event = 10
   (a) odd number

   Favourable = 5 *i.e.* 1, 3, 5, 7, 9 Probability = $\frac{5}{10} = \frac{1}{2}$

   (b) getting a prime number

   favourable = 2, 3, 5, 7, 2 Probability = $\frac{4}{10} = \frac{2}{5}$

   (c) Events multiple of 2

   favourable = 5 Probability = $\frac{5}{10} = \frac{1}{2}$

Check Your Mental Math I.Q.

1. Total number of event = 52

(a) a diamond choosen

Favourable = 13 Probability = $\frac{13}{52} = \frac{1}{4}$

(b) a king choosen

Favourable = 4 Probability = $\frac{4}{52} = \frac{1}{13}$

(c) a black 6 choosen

favourable = 2 Probability = $\frac{2}{52} = \frac{1}{26}$

(d) a 8 of heart is choosen

Favourable = 1 Probability = $\frac{1}{52}$

2. Total number of event = 6

(a) Prime number

Favourable event=3, 2, 5 *i.e.*3 Probablity = $\frac{3}{6} = \frac{1}{2}$

(b) a number between 3 and 6

Varourable = 4, 5 *i.e.* 2 Probablity = $\frac{2}{6} = \frac{1}{3}$

(c) odd numbr between 4 and 6

Favourable 5 *i.e.* 1 Probablity = $\frac{1}{6}$

3. Total number of marbles = 5 + 7 = 12

(a) a red marbel

Favourable = 5 Probablity = $\frac{5}{12}$

(b) a blue marble

Favourable = 7 Probablity = $\frac{7}{12}$

4. Total bulbs = 1200

remaining = 1200 – 24 = 1176

probability of remaining (good) = $\frac{1176}{1200} = \frac{147}{150}$

Multiple Choice Questions (M.C.Q.)

1. (a) Total event 6

   Favourable = 1 Probablity = $\frac{1}{6}$

2. (b) Total events = 12

   Favourable = 7 Probablity = $\frac{7}{12}$

3. (c) Total event = 52

   Favourable = 4 Probablity = $\frac{4}{52} = \frac{1}{13}$

4. (d) black ball
   Total event = 15

   Favourable = 9 Probablity = $\frac{9}{15} = \frac{3}{5}$

21 Introduction to Graph

Check Your Mental Math I.Q.

A. 1. True 2. True 3. True
4. False 5. True

B. 1. different 2. second
3. (0, 0) 4. four
5. x-axis

Multiple Choice Question (M.C.Q.)

1. (b) and (c) 2. (b) (4, 0) 3. (a) Origin
4. (a) (0, 3) 5. (a) origin